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Electroweak contributions to $e^+e^- \to W^+W^-b\bar{b}$ in the $t\bar{t}$ resonance region

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I Top-pair production at linear colliders near threshold

Future linear colliders (ILC/CLIC)
with $\sqrt{s} \gtrsim 2m_t \approx 350$ GeV $\rightsquigarrow$ produce $t\bar{t}$ pairs:
clean initial state of $e^+e^- \to t\bar{t}$ allows threshold scans with $\sqrt{s} \sim 2m_t$

$\hookrightarrow$ Precise determination of the top mass m_t, the width Γ_t, the Yukawa coupling λ_t without the uncertainties/ambiguities of hadron colliders. Martinez, Miquel '02

Need also precise theoretical prediction
$\Rightarrow \delta\sigma/\sigma \sim$ 2–3% ($\delta\sigma \sim 5$ fb below threshold)
$\rightsquigarrow$ Important input for electroweak precision observables!

QCD corrections are known (almost) up to NNNLO order,
but electroweak (EW) NLO hard contributions are missing!

Also: decay $t\bar{t} \to (bW^+)\,(\bar{b}W^-)$ is an EW effect
$\Rightarrow$ describe $t\bar{t}$ production in terms of the more physical process $e^+e^- \to W^+W^-b\bar{b}$
and allow for invariant-mass cuts on reconstructed $t, \bar{t}$.

Perturbative expansion: NRQCD

Decay $t \to bW^+$ with $\Gamma_t \approx 1.5\,\text{GeV} \gg \Lambda_{\text{QCD}} \Rightarrow t\bar{t}$ is perturbative at threshold.

Bigi, Dokshitzer, Khoze, Kühn, Zerwas '86

But top quarks move slowly at threshold: $v = \sqrt{1 - \frac{4m_t^2}{s}} \sim \alpha_s \ll 1$

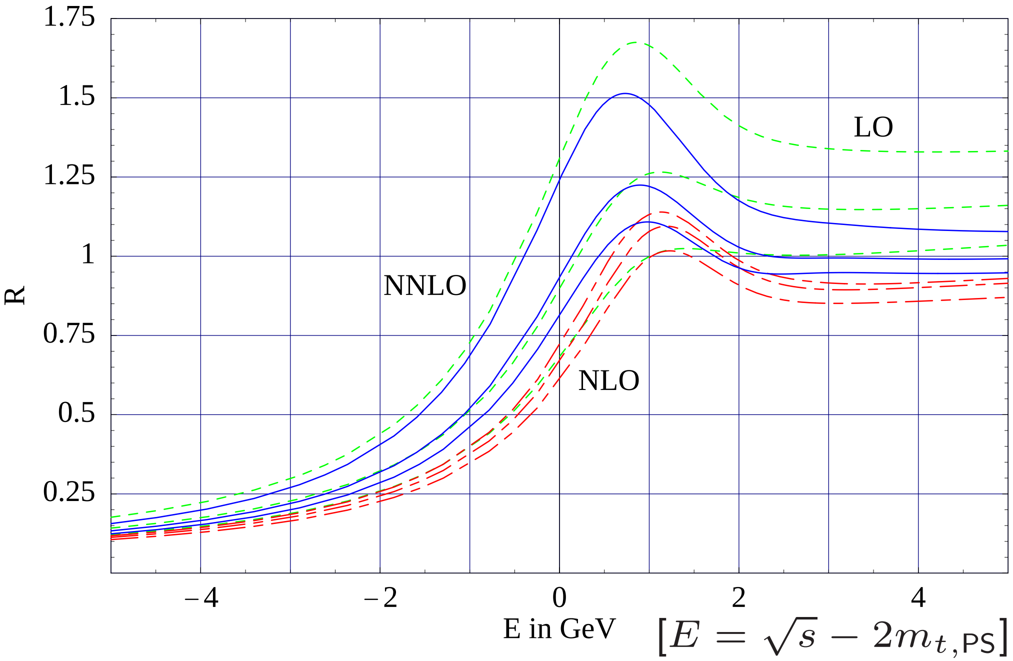
$\hookrightarrow$ sum $\left(\frac{\alpha_s}{v}\right)^n$ from "Coulomb gluons" to all orders:

$$R = \frac{\sigma_{t\bar{t}}}{\sigma_{\mu^+\mu^-}} = v \sum_n \left(\frac{\alpha_s}{v}\right)^n \left(\{1\}_{\text{LO}} + \{\alpha_s, v\}_{\text{NLO}} + \{\alpha_s^2, \alpha_s v, v^2\}_{\text{NNLO}} + \dots \right)$$

Further RGE improvement by summing also $(\alpha_s \ln v)^m$ to all orders: LL, NLL, ...

Status of QCD corrections

- NNLO QCD corrections
  Hoang, Teubner '98–'99; Melnikov, Yelkhovsky '98; Yakovlev '98; Beneke, Signer, Smirnov '99 [see plot]; Nagano, Ota, Sumino '99; Penin, Pivovarov '98–'99
- NNLO & (partial) NNLL
  Hoang, Manohar, Stewart, Teubner '00–'01; Hoang '03; Pineda, Signer '06
- (partial) NNNLO
  Beneke, Kiyo, Schuller '05–'08 [+ contributions from Kiyo, Seidel, Steinhauser '08; Anzai, Kiyo, Sumino '09; Smirnov, Smirnov, Steinhauser '09–'10]



$[E = \sqrt{s} - 2m_{t,\text{PS}}]$

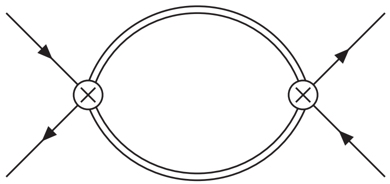
Effective field theory (EFT) for pair production of unstable particles near threshold

Beneke, Chapovsky, Khoze, Signer, Stirling, Zanderighi '01–'04;
Actis, Beneke, Falgari, Schwinn, Signer, Zanderighi '07–'08

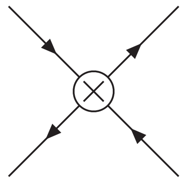
- Non-relativistic power counting: $\boxed{\alpha_s^2 \sim \alpha_{\text{ew}} \sim \frac{\Gamma_t}{m_t} \sim v^2 \approx \delta = \frac{s}{4m_t^2} - 1}$
- Integrate out hard modes $\sim m_t \rightsquigarrow$ EFT with potential (nearly on-shell) top quarks.
- Extract cross section $e^+e^- \to W^+W^-b\bar{b}$ from appropriate cuts of the $e^+e^- \to e^+e^-$ forward-scattering amplitude:

$$i\mathcal{A} = \underbrace{\sum_{k,l} \int \mathrm{d}^4x \, \langle e^+e^- | T\big[i\mathcal{O}_p^{(k)\dagger}(0)\, i\mathcal{O}_p^{(l)}(x)\big] | e^+e^- \rangle}_{\text{resonant contributions}} + \underbrace{\sum_k \langle e^+e^- | i\mathcal{O}_{4e}^{(k)}(0) | e^+e^- \rangle}_{\text{non-resonant contributions}}$$

resonant contributions
with $t\bar{t}$ production operators $\mathcal{O}_p^{(k)}$



non-resonant contributions



correspond to full-theory
$e^+e^- \to e^+e^-$ with $\Gamma_t = 0$

$\Rightarrow$ Potential (+ soft ...) corrections to resonant diagrams within EFT

$\Rightarrow$ Hard corrections to matching coefficients of operators $\mathcal{O}_p^{(k)}$ and $\mathcal{O}_{4e}^{(k)}$.

Electroweak effects at LO

- Replacement rule $E = \sqrt{s} - 2m_t \to E + i\Gamma_t$ Fadin, Khoze '87

Electroweak effects at NLO

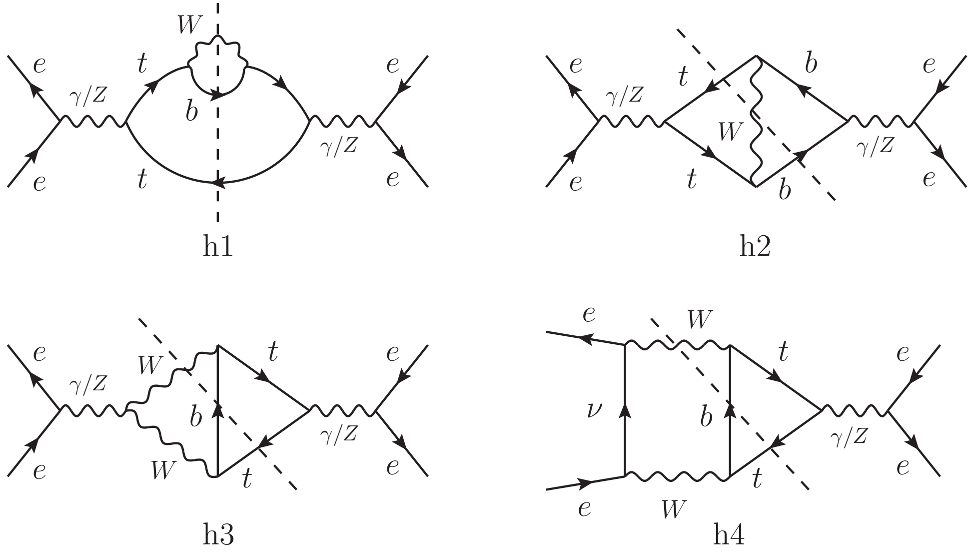
- Exchange of a "Coulomb photon": trivial extension of QCD corrections, available
- Gluon exchange between $t, \bar{t}$ and their decay products: these contributions cancel at NLO in the total cross section, Fadin, Khoze, Martin '94; Melnikov, Yakovlev '94
they are negligible if the top invariant-mass cuts are loose enough.
- **Hard corrections** to the matching coefficient of the **non-resonant operator** $\mathcal{O}_{4e}^{(k)}$
$\hookrightarrow$ *topic of this talk!*

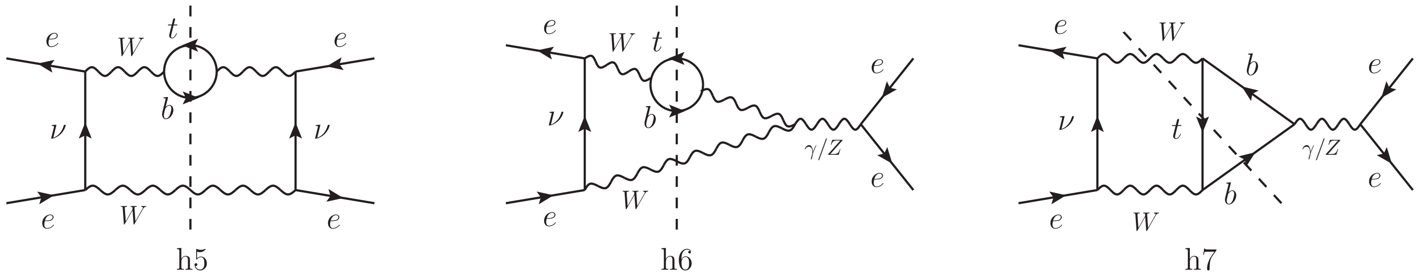
The resonant **NNLO** corrections involve "finite-width divergences" $\propto \alpha_s \dfrac{\Gamma_t}{\epsilon}$ (in dim. reg.). These must be cancelled by EW NNLO hard contributions.
$\hookrightarrow$ Motivation for calculating EW hard corrections (starting at NLO . . .).

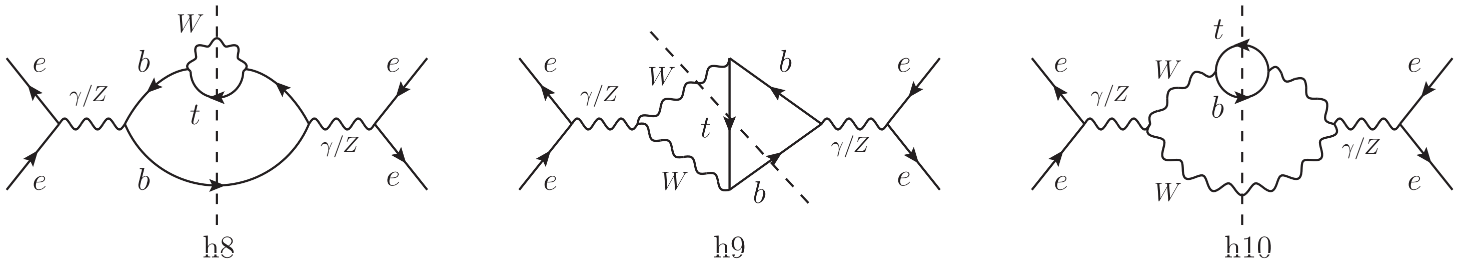
II Evaluation of electroweak NLO hard contributions

Hard corrections at NLO:

- cuts through $bW^+\bar{t}$ (see diagrams) and $\bar{b}W^-t$ (not shown) in the 2-loop forward-scattering amplitude
- correspond to tree-level processes $e^+e^- \to bW^+\bar{t}$ and $e^+e^- \to \bar{b}W^-t$
- suppressed w.r.t. LO ($\sim v$) by $\alpha_{\text{ew}}/v \sim \alpha_s$
- expansion in $\delta = \frac{s}{4m_t^2} - 1$
  $\hookrightarrow$ at NLO: $s = 4m_t^2$
  [We keep the full s-dependence outside the phase-space integral]
- hard region: $\Gamma_t = 0$.
  [Divergence at $p_t^2 = m_t^2$ in diagram h1 regulated dimensionally $\rightsquigarrow$ finite *negative* contribution]







[symmetric diagrams not shown]

Form of hard contributions

With the reconstructed top momentum $p_t = p_b + p_{W^+}$ (only h1–h4 actually have this top), the contributions of diagrams h1–h10 (for $s = 4m_t^2$) are of the form:

$$H_i = \int_{\Delta^2}^{m_t^2} \mathrm{d}p_t^2 \, (m_t^2 - p_t^2)^{1/2-\epsilon} \, \hat{h}_i\left(\frac{p_t^2}{m_t^2}, \frac{M_W^2}{m_t^2}\right)$$

with $\Delta^2 = M_W^2$ for the total cross section.

[The phase-space factor $(m_t^2 - p_t^2)^{1/2-\epsilon}$ in dim. reg. regularizes the end-point singularity for h1.]

Invariant-mass cuts

Restrict invariant masses of the reconstructed $t, \bar{t}$: $\left|\sqrt{p_{t,\bar{t}}^2} - m_t\right| \leq \Delta M$

where the total cross section is reached for $\Delta M = \max\{m_t - M_W, \sqrt{s} - m_t - M_W\}$

(also for full process $e^+e^- \to W^+W^-b\bar{b}$ with general s)

$\hookrightarrow$ hard contributions with $\Delta^2 = m_t^2 - \Lambda^2$ where $\Lambda^2 = (2m_t - \Delta M)\Delta M \leq m_t^2 - M_W^2$.

We focus on loose cuts with $\Lambda^2 \gg m_t\Gamma_t$ (typical offshellness of potential top quarks), corresponding to $\Delta M \gg \Gamma_t \rightsquigarrow$ no cut needed for potential contributions.

In contrast: for tight cuts with $\Lambda^2 \lesssim m_t\Gamma_t$ or $\Delta M \lesssim \Gamma_t \rightsquigarrow$ hard contributions are absent and potential contributions need to be cut.

III Results & comparisons

EW NLO hard contributions: numeric integration over p_t^2 (and over one angle for some diagrams), the integrand is an analytic function of p_t^2/m_t^2 and M_W^2/m_t^2, the cut-dependence enters in the integration limit.

Parameters: $m_t = 172\,\text{GeV}$, $\Gamma_t = \Gamma_t^{\text{tree}} = 1.46550\,\text{GeV}$, on-shell (pole) masses, α and $\sin^2\theta_{\text{w}}$ from G_{F}, M_W, M_Z

Comparison to recent alternative approach (HRR) Hoang, Reißer, Ruiz-Femenía '10

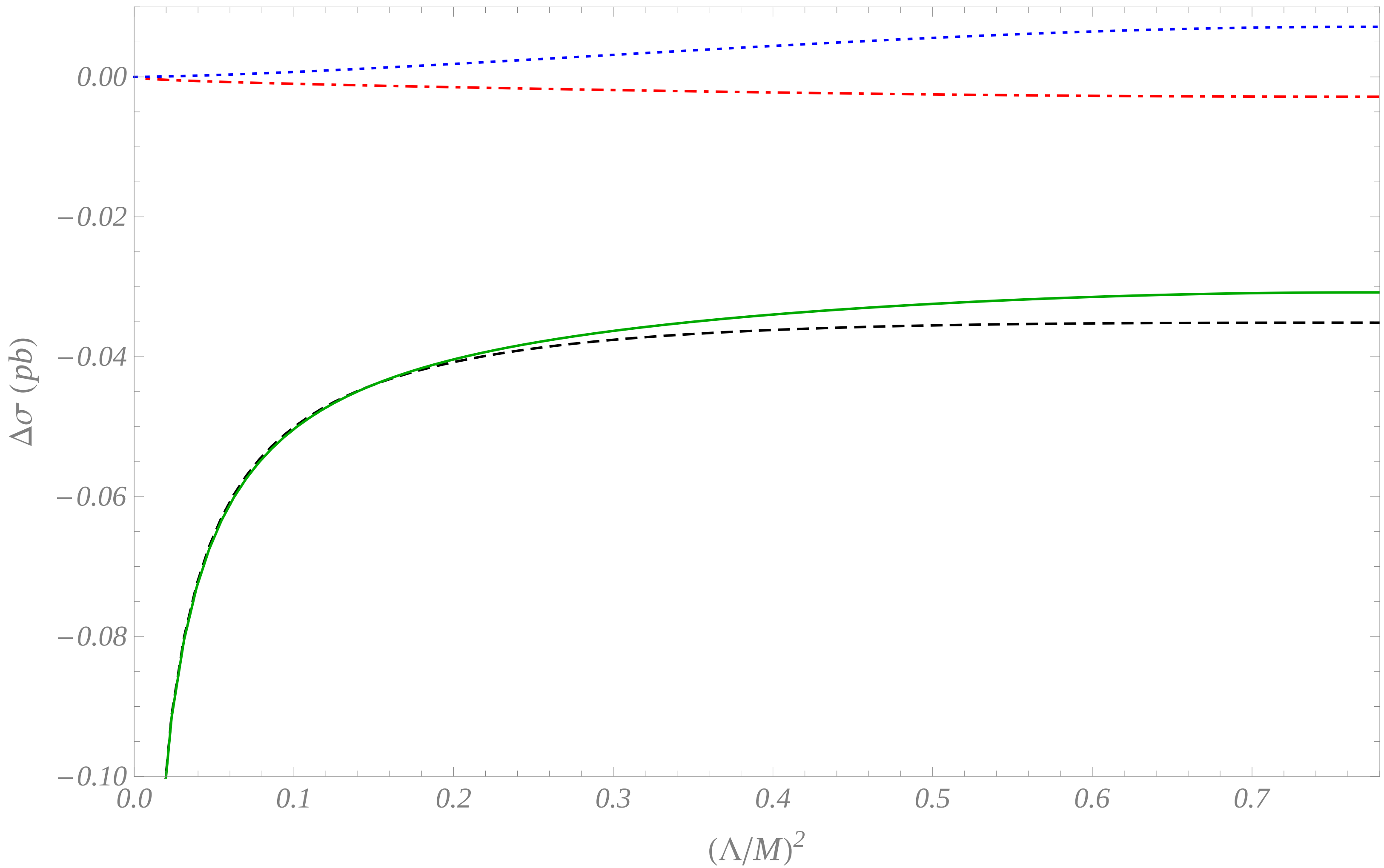
- Here QCD & EW contributions expanded for moderate invariant-mass cuts $m_t\Gamma_t \ll \Lambda^2 \ll m_t^2$, in particular $15\,\text{GeV} \leq \Delta M \leq 35\,\text{GeV}$
  $\hookrightarrow$ our result is also valid for larger cuts up to the total cross section.
- EW contributions match expansion of our result in $(\Lambda/m_t)^n$, including orders $n = -1$ from h1 and $n = 1$ from "double-resonant" diagrams h1–h4.
- HRR had to neglect the non-t diagrams h5–h10 (which start with $(\Lambda/m_t)^3$), these "single-resonant" contributions are systematically included in our calculation.

Comparison to MadGraph/MadEvent/MadAnalysis (MG) Alwall et al. '07

$\hookrightarrow$ generated 10^4 events for $e^+e^- \to W^+W^-b\bar{b}$, analyzed cut-dependence

Hard corrections: contributions of the diagrams

contribution to cross section as a function of the invariant-mass cut Λ



total cross section reached for $(\Lambda/M)^2 = 0.78$

all hard diagrams: solid-green

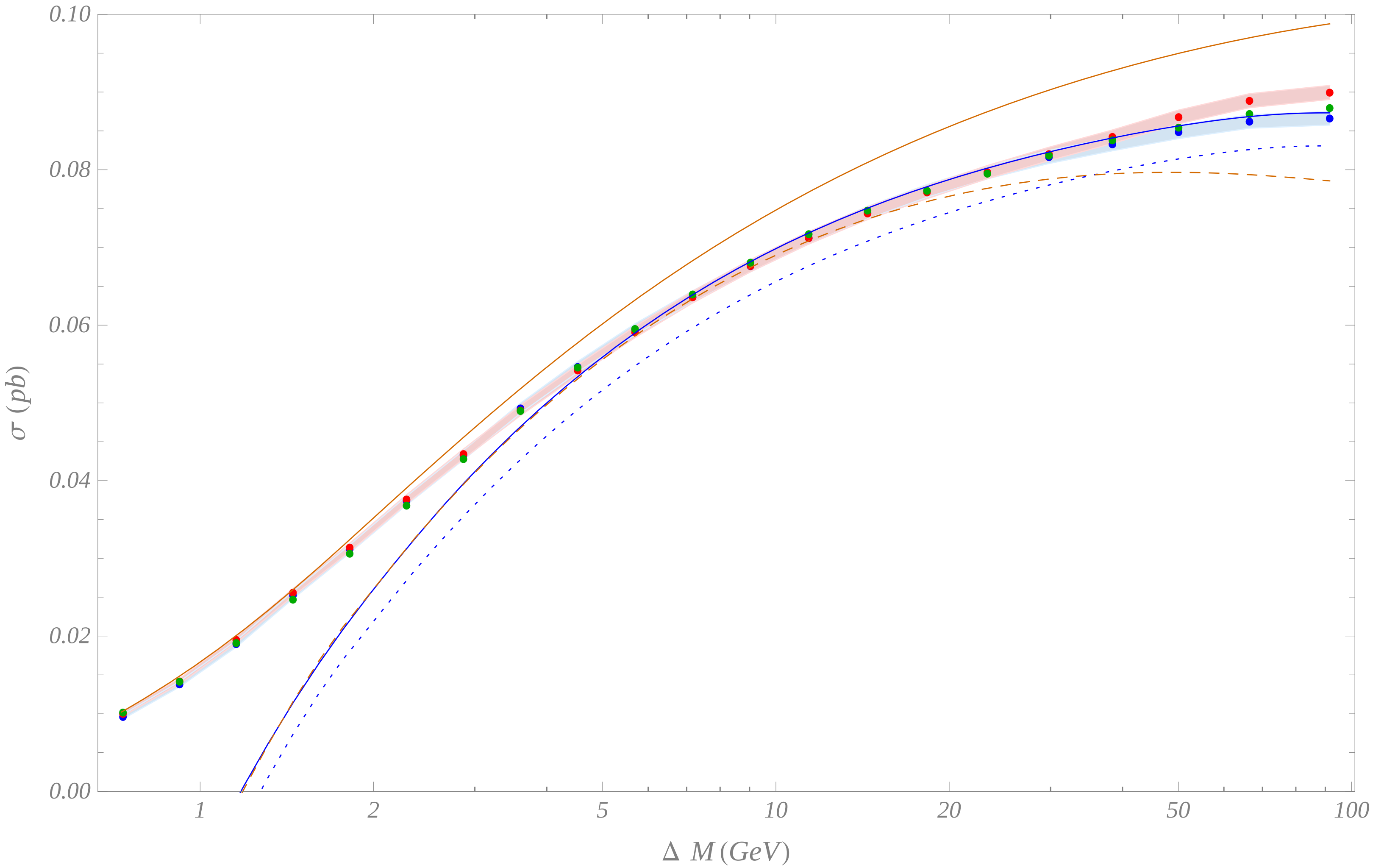
dominant diagram h1: dashed-black

other double-resonant diagrams h2–h4: dashed-dotted-red

single-resonant diagrams h5–h10: dotted-blue

EW contributions: cut-dependence at threshold

cross section (for $\alpha_s = 0$) at threshold ($s = 4m_t^2$) as a function of the invariant-mass cut ΔM



total cross section reached for $\Delta M = 91.6$ GeV

MG points (with statistical error bands): full, without Higgs, only t- or $\bar{t}$-diagrams

Our result: EW NLO hard + LO (dashed-blue) / NNLO (solid-blue) potential contributions

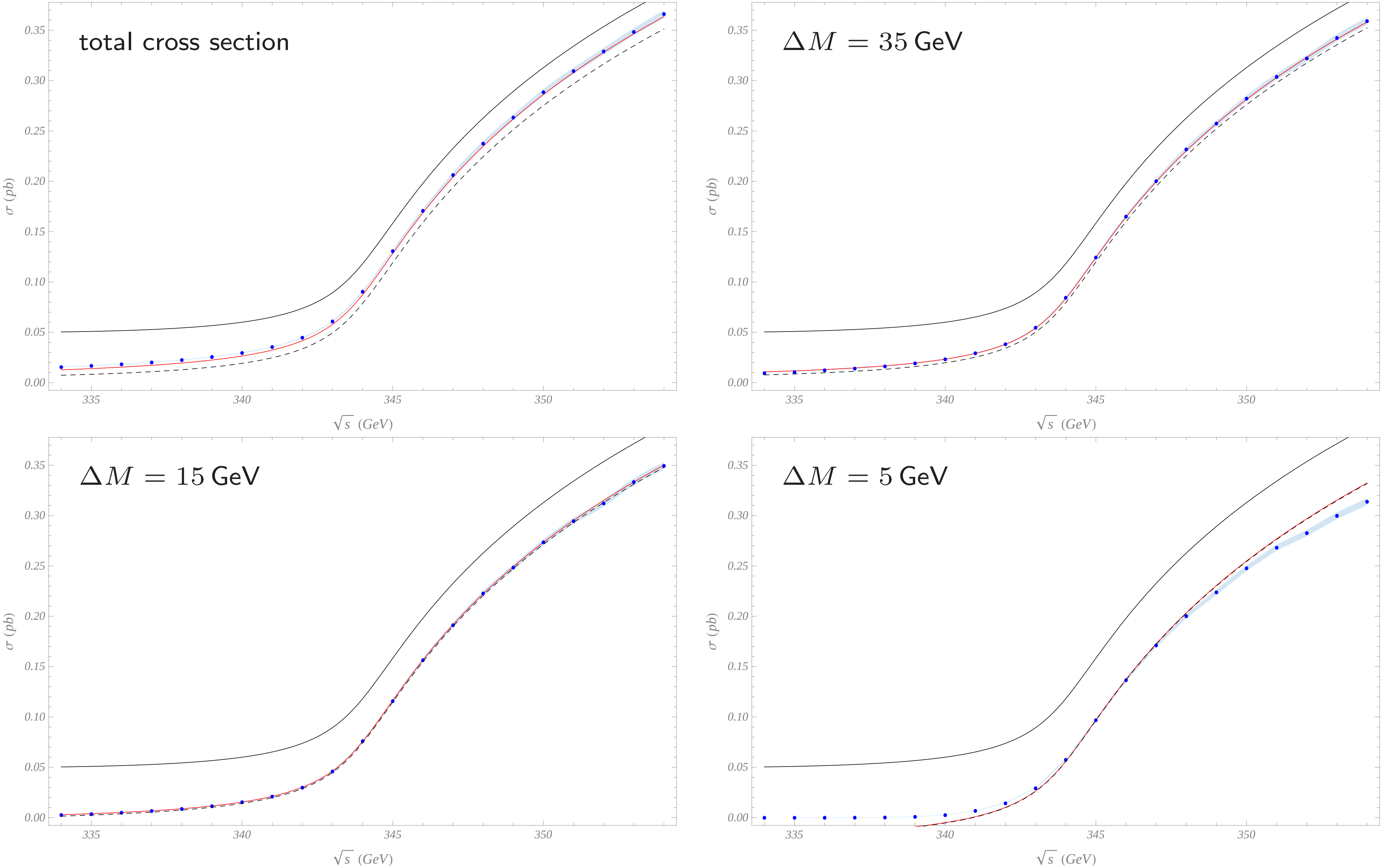
$\hookrightarrow$ good agreement with MG for loose cuts $\Delta M \gtrsim 5$ GeV

Cut potential region (LO): solid-brown $\Rightarrow$ good agreement with MG for tight cuts $\Delta M \lesssim 1$ GeV

HRR result: dashed-brown $\Rightarrow$ agrees with our result for small ΔM

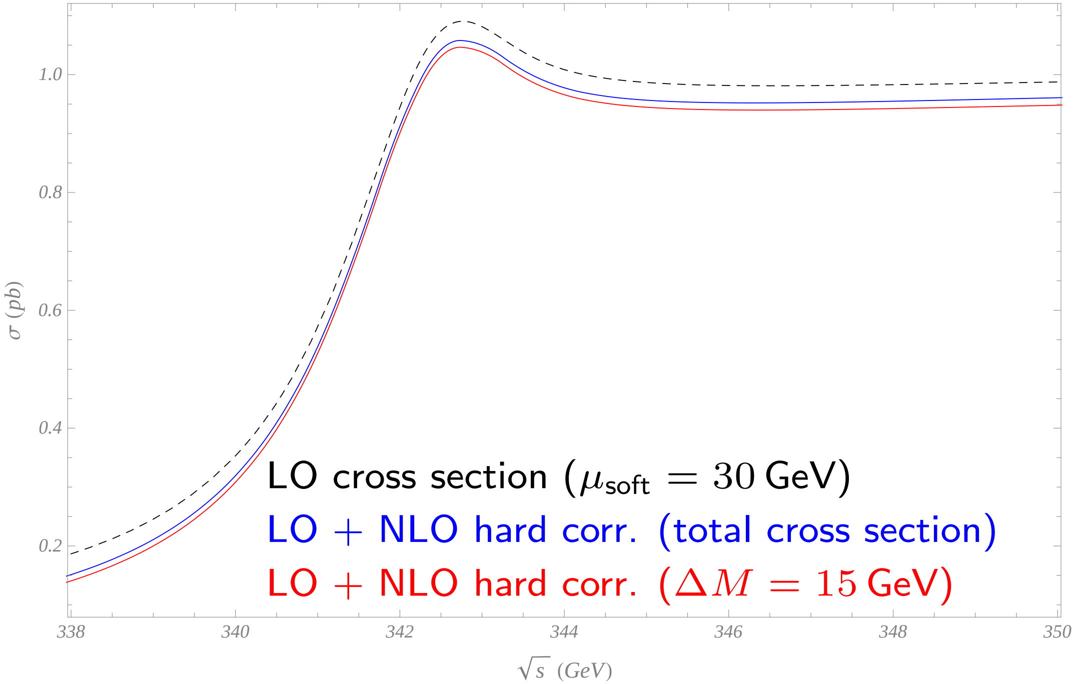
EW contributions: energy-dependence for different cuts

cross section (for $\alpha_s = 0$) as a function of the center-of-mass energy $\sqrt{s}$



MG (full) points & error band, our result, HRR result (dashed), only potential contributions (solid)

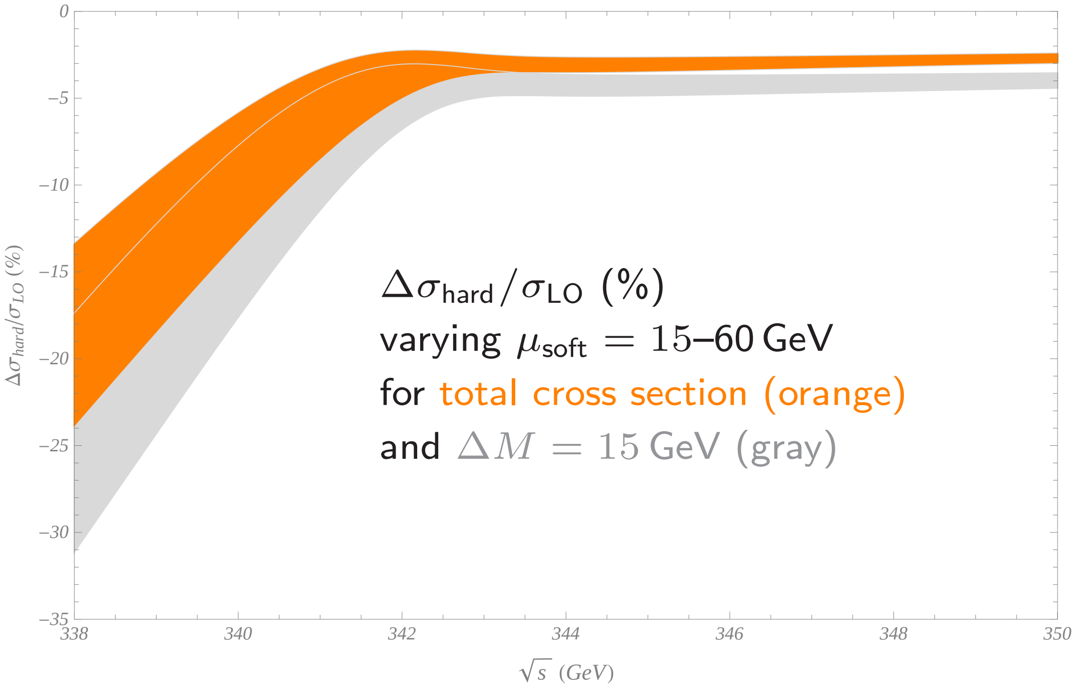
Full cross section with QCD & EW contributions



QCD contributions with $\alpha_s^{\overline{\text{MS}}}(\mu_{\text{soft}})$ (obtained from $\alpha_s^{\overline{\text{MS}}}(M_Z) = 0.118$)

LO cross section (potential QCD & EW contributions, dashed-black) and including our **EW NLO hard contributions** (solid-colored)

[NLO QCD contributions not shown]



Relative correction (in %) of EW NLO hard contribution w.r.t. LO cross section

IV Conclusions & outlook

Electroweak contributions to $e^+e^- \to W^+W^-b\bar{b}$ in the $t\bar{t}$ resonance region

- NLO contribution completed by EW non-resonant (hard) contributions for total cross section and with top invariant-mass cuts
- correction of $\sim$ -30 fb (-3% above and much more below threshold) for total cross section, even more with invariant-mass cuts
- good agreement with MadGraph for loose cuts
- good agreement with Hoang–Reißer–Ruiz-Femenía result for small cuts

$\hookrightarrow$ can be added to existing QCD results to improve accuracy of theoretical prediction

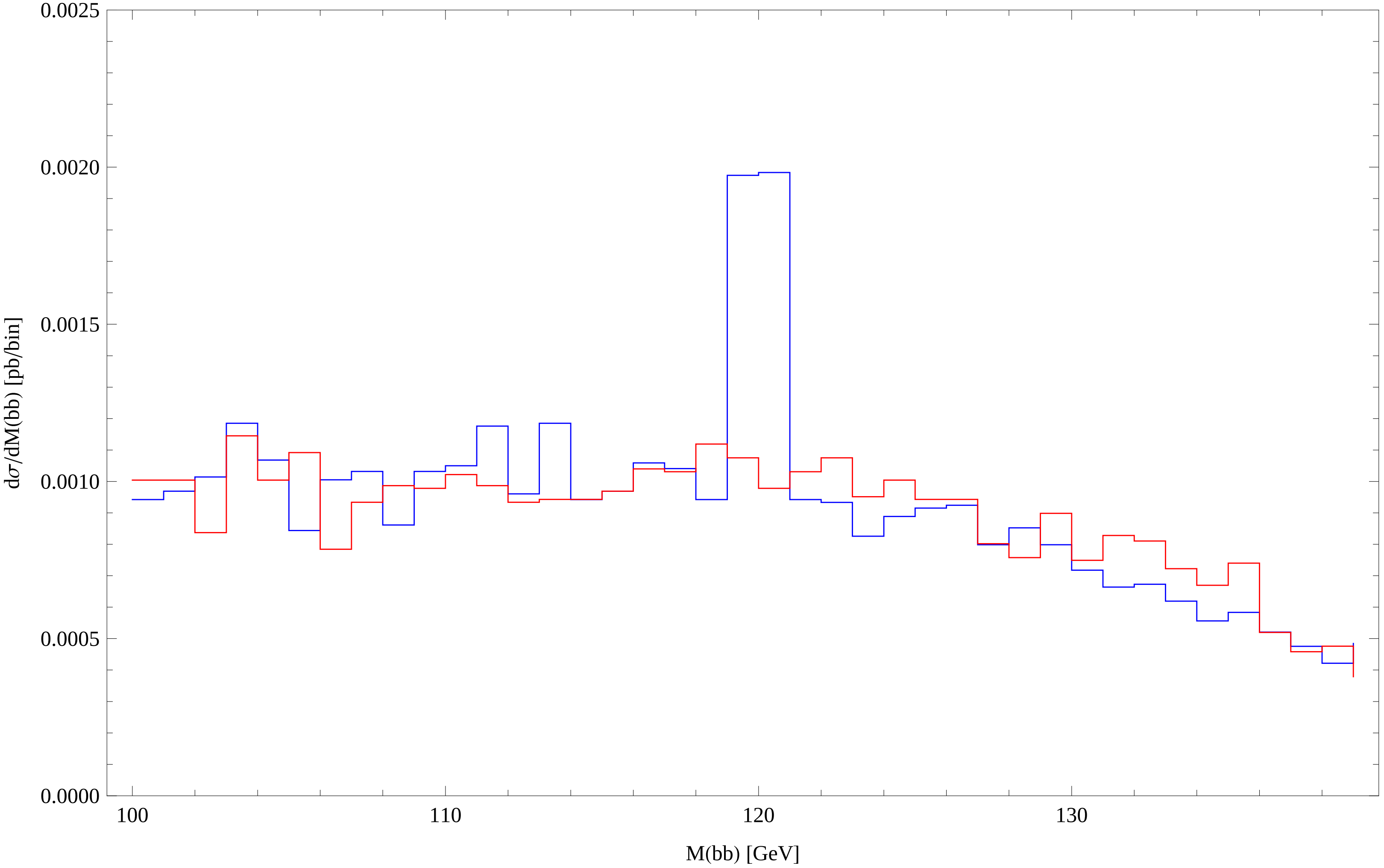
Future improvements

- add initial-state radiation and convolution with electron distribution functions
- evaluate leading EW NNLO contributions $\Rightarrow$ cancel finite-width divergences

Extra slides

Contribution of Higgs diagrams

distribution with respect to the invariant mass of $b\bar{b}$



MG points: full distribution and without Higgs diagrams

↪ Higgs peak in $d\sigma/dM(b\bar{b})$ from diagrams $e^+e^- \to W^+W^-(H \to b\bar{b})$