International Workshop on Linear Colliders 2010
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Electroweak NLO corrections to top threshold production

– more precisely –

Electroweak non-resonant NLO corrections to $e^+e^- \to W^+W^-b\bar{b}$ in the $t\bar{t}$ resonance region

Bernd Jantzen

RWTH Aachen University

In collaboration with **Martin Beneke** and **Pedro Ruiz-Femenía**

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I Top-pair production at linear colliders near threshold

Future linear colliders (ILC/CLIC)

with $\sqrt{s} \gtrsim 2m_t \approx 350\,$GeV $\rightsquigarrow$ produce $t\bar{t}$ pairs:

clean initial state of $e^+e^- \to t\bar{t}$ allows threshold scans with $\sqrt{s} \sim 2m_t$

$\hookrightarrow$ Precise determination of the top mass m_t, the width Γ_t, the Yukawa coupling λ_t without the uncertainties/ambiguities of hadron colliders. Martinez, Miquel '02

Need also precise theoretical prediction

$\Rightarrow$ $\delta\sigma/\sigma \sim$ 2–3% ($\delta\sigma \sim$ 5 fb below threshold)

$\rightsquigarrow$ Important input for electroweak precision observables!

QCD corrections are known (almost) up to NNNLO order,

but electroweak (EW) non-resonant NLO contributions were missing!

Also: decay $t\bar{t} \to (bW^+)\,(\bar{b}W^-)$ is an EW effect

$\Rightarrow$ describe $t\bar{t}$ production in terms of the more physical process $e^+e^- \to W^+W^-b\bar{b}$

and allow for invariant-mass cuts on reconstructed $t, \bar{t}$.

Perturbative expansion: NRQCD

Decay $t \to bW^+$ with $\Gamma_t \approx 1.5\,\text{GeV} \gg \Lambda_{\text{QCD}} \Rightarrow t\bar{t}$ is perturbative at threshold.
Bigi, Dokshitzer, Khoze, Kühn, Zerwas '86

But top quarks move slowly near threshold: $v = \sqrt{1 - \frac{4m_t^2}{s}} \sim \alpha_s \ll 1$

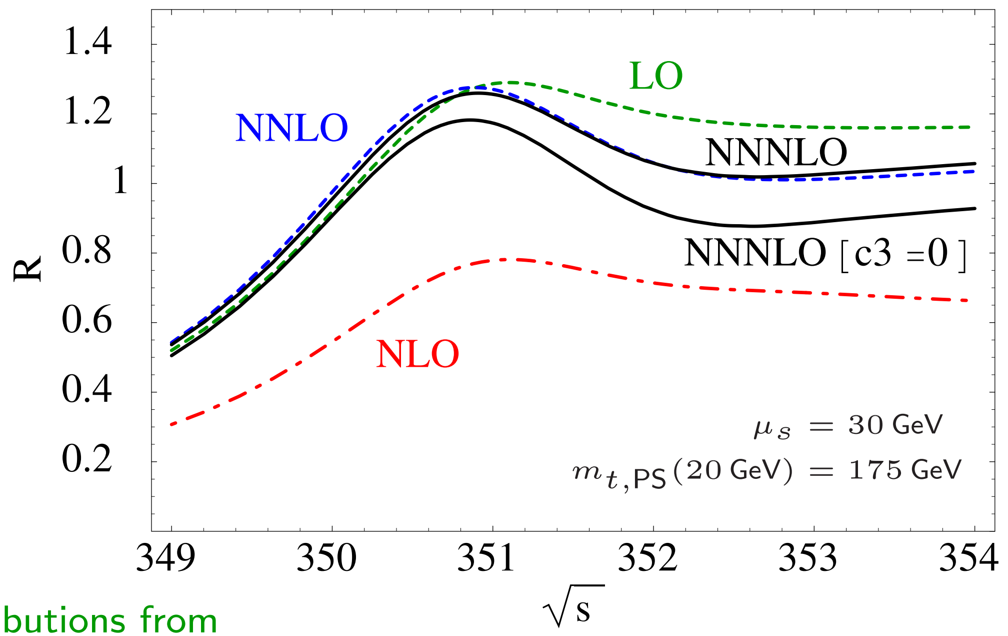
$\hookrightarrow$ sum $\left(\frac{\alpha_s}{v}\right)^n$ from "Coulomb gluons" to all orders:

$$R = \frac{\sigma_{t\bar{t}}}{\sigma_{\mu^+\mu^-}} = v \sum_n \left(\frac{\alpha_s}{v}\right)^n \left(\{1\}_{\text{LO}} + \{\alpha_s, v\}_{\text{NLO}} + \{\alpha_s^2, \alpha_s v, v^2\}_{\text{NNLO}} + \ldots\right)$$

Further RGE improvement by summing also $(\alpha_s \ln v)^m$ to all orders: LL, NLL, ...

Status of QCD corrections

- NNLO QCD corrections
  Hoang, Teubner '98–'99; Melnikov, Yelkhovsky '98; Yakovlev '98; Beneke, Signer, Smirnov '99; Nagano, Ota, Sumino '99; Penin, Pivovarov '98–'99
- NNLO & (partial) NNLL
  Hoang, Manohar, Stewart, Teubner '00–'01; Hoang '03; Pineda, Signer '06
- (partial) NNNLO
  Beneke, Kiyo, Schuller '05–'08 $\rightsquigarrow$ see figure [+ contributions from Kiyo, Seidel, Steinhauser '08; Anzai, Kiyo, Sumino '09; Smirnov, Smirnov, Steinhauser '09–'10]

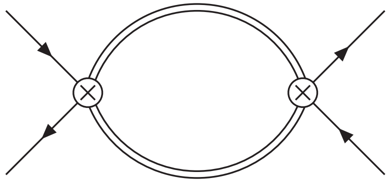


Effective field theory (EFT) for pair production of unstable particles near threshold

Beneke, Chapovsky, Khoze, Signer, Stirling, Zanderighi '01–'04;
Actis, Beneke, Falgari, Schwinn, Signer, Zanderighi '07–'08

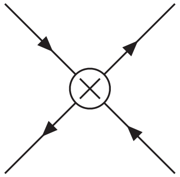
- Non-relativistic power counting: $\alpha_s^2 \sim \alpha_{\mathsf{EW}} \sim \frac{\Gamma_t}{m_t} \sim v^2 \approx \delta = \frac{s - 4m_t^2}{4m_t^2}$
- Integrate out hard modes $\sim m_t \rightsquigarrow$ EFT with potential (nearly on-shell) top quarks.
- Extract cross section $e^+e^- \to W^+W^-b\bar{b}$ from appropriate cuts of the $e^+e^- \to e^+e^-$ forward-scattering amplitude:

resonant contributions



with $t\bar{t}$ production operators

non-resonant contributions



correspond to full-theory $e^+e^- \to e^+e^-$ with $\Gamma_t = 0$

$\Rightarrow$ Potential corrections to resonant diagrams within EFT

$\Rightarrow$ Hard corrections to matching coefficients of operators

Electroweak effects at LO

- Replacement rule $E = \sqrt{s} - 2m_t \to E + i\Gamma_t$ Fadin, Khoze '87

Electroweak effects at NLO

- Exchange of a "Coulomb photon": trivial extension of QCD corrections (available)
- Gluon exchange between top quarks and their decay products: these contributions cancel at NLO in the total cross section, Fadin, Khoze, Martin '94; Melnikov, Yakovlev '94 they are negligible if the top invariant-mass cuts are loose enough.
- **Non-resonant (hard) corrections $\rightsquigarrow$ topic of this talk!**

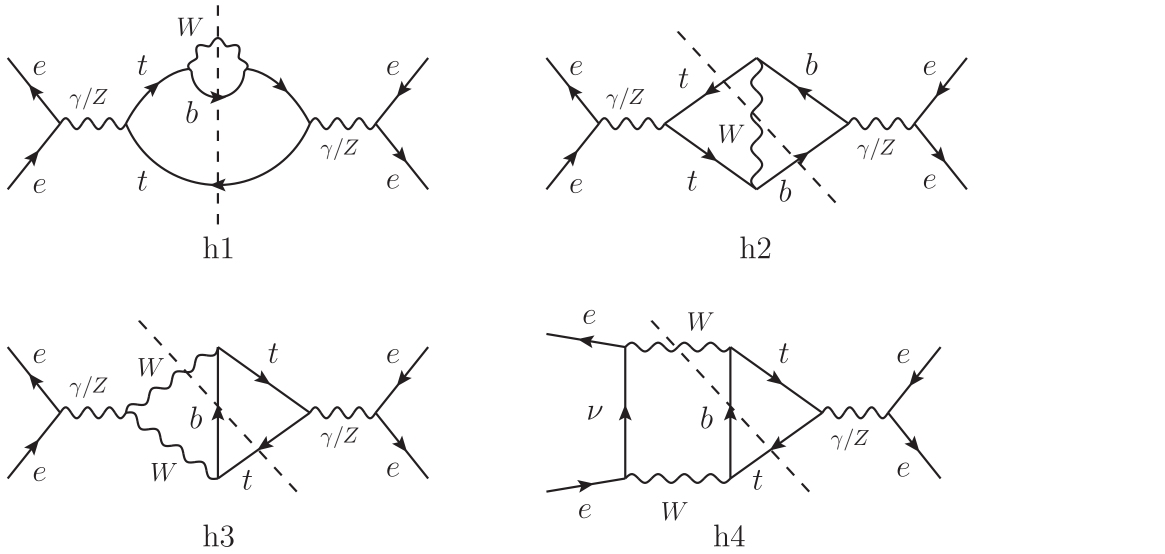
The resonant **NNLO** corrections involve "finite-width divergences" $\propto \alpha_s \dfrac{\Gamma_t}{\epsilon}$ (in dim. reg.). These must be cancelled by EW non-resonant NNLO contributions.
$\hookrightarrow$ Motivation for calculating EW non-resonant corrections (starting at NLO ...).

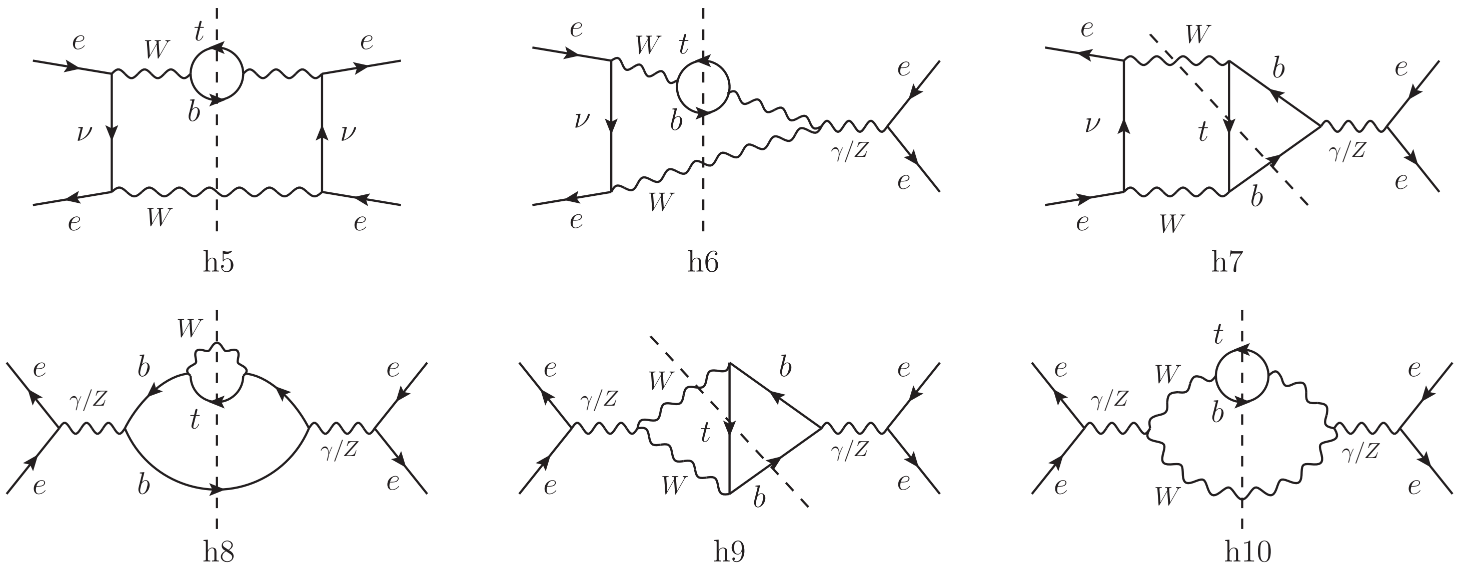
II Evaluation of EW non-resonant NLO contributions

Non-resonant corrections at NLO:

- cuts through $bW^+\bar{t}$ (see diagrams) and $\bar{b}W^-t$ (not shown) in the 2-loop forward-scattering amplitude
- correspond to tree-level processes $e^+e^- \to bW^+\bar{t}$ and $e^+e^- \to \bar{b}W^-t$
- suppressed w.r.t. LO ($\sim v$) by $\alpha_{\mathsf{EW}}/v \sim \alpha_s$
- at NLO: $s = 4m_t^2$
- hard region: $\Gamma_t = 0$

  [Divergence at $p_t^2 = m_t^2$ in diagram h1 regularized dimensionally $\rightsquigarrow$ finite *negative* contribution]





[symmetric diagrams not shown]

Form of non-resonant contributions

With the reconstructed top momentum $p_t = p_b + p_{W^+}$ (only h1–h4 have this top), the contributions of diagrams h1–h10 (for $s = 4m_t^2$) are of the form:

$$\int_{\Delta^2}^{m_t^2} \mathrm{d}p_t^2 \, (m_t^2 - p_t^2)^{1/2-\epsilon} \, H_i\!\left(\frac{p_t^2}{m_t^2}, \frac{M_W^2}{m_t^2}\right)$$

with $\Delta^2 = M_W^2$ for the total cross section.

[In dim. reg. the phase-space factor $(m_t^2 - p_t^2)^{1/2-\epsilon}$ regularizes the end-point singularity for h1.]

Invariant-mass cuts

Restrict invariant masses $M_{t,\bar{t}}$ of the reconstructed $t, \bar{t}$: $\left|M_{t,\bar{t}} - m_t\right| \leq \Delta M_t$

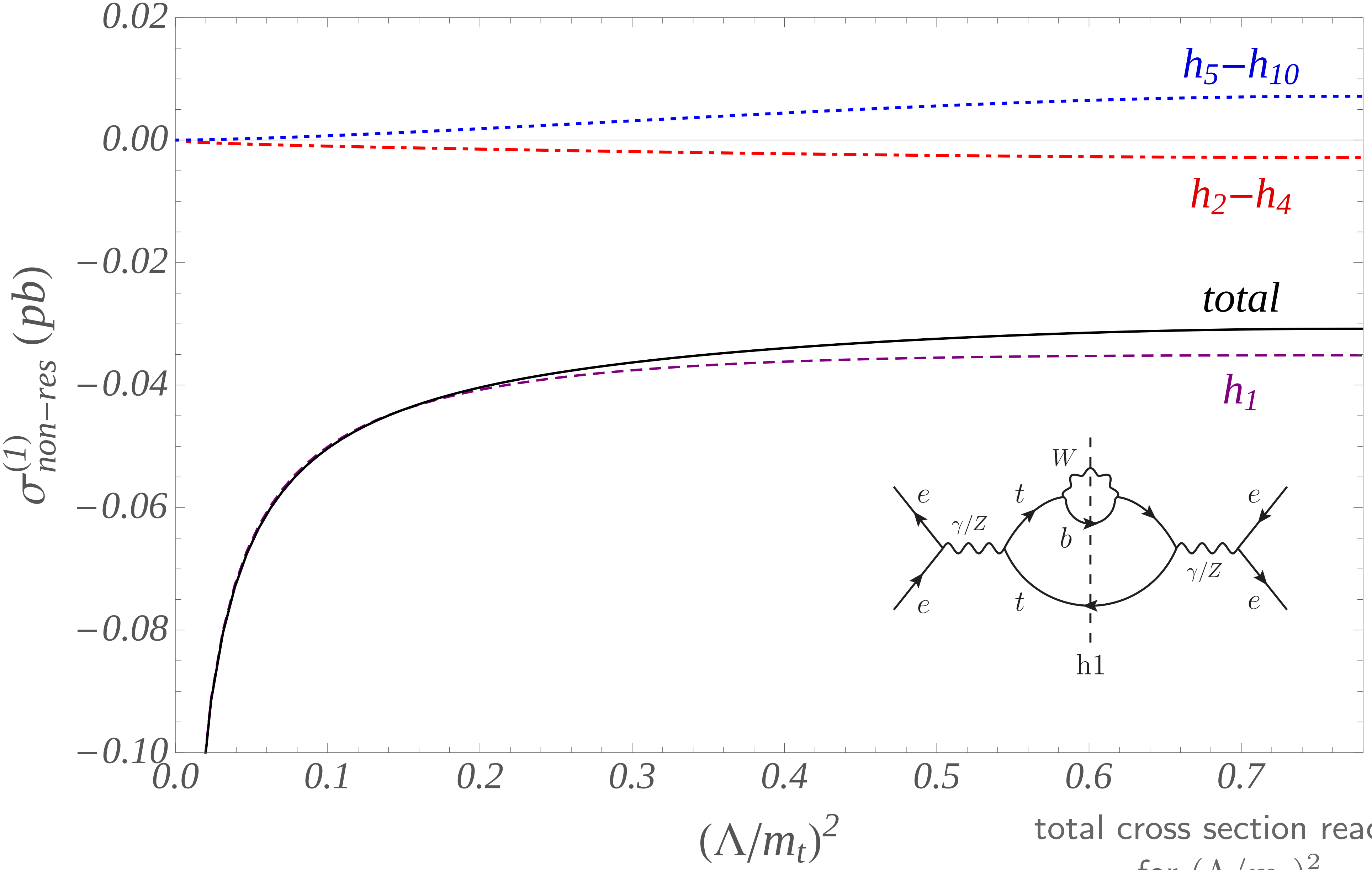
$\hookrightarrow$ lower integration limit $\Delta^2 = m_t^2 - \Lambda^2$ where $\Lambda^2 = (2m_t - \Delta M_t)\Delta M_t \leq m_t^2 - M_W^2$.

We focus on loose cuts with $\Lambda^2 \gg m_t\Gamma_t$ = typical offshellness of potential top quarks (corresponding to $\Delta M_t \gg \Gamma_t$) $\rightsquigarrow$ no cut needed for resonant contributions.

In contrast: for tight cuts with $\Lambda^2 \lesssim m_t\Gamma_t$ or $\Delta M_t \lesssim \Gamma_t$ $\rightsquigarrow$ non-resonant contributions are absent and resonant contributions need to be cut.

Non-resonant corrections: contributions of the diagrams

contribution to cross section as a function of the invariant-mass cut Λ



total cross section reached for $(\Lambda/m_t)^2 = 0.78$

III Results & comparisons

Parameters for non-resonant contributions: on-shell (pole) mass $m_t = 172\,\text{GeV}$, $\Gamma_t = \Gamma_t^{\text{tree}} = 1.46550\,\text{GeV}$, α and $\sin^2\theta_{\text{w}}$ from G_{F}, M_W, M_Z

Comparison to recent alternative approach (HRR) Hoang, Reißer, Ruiz-Femenía '10

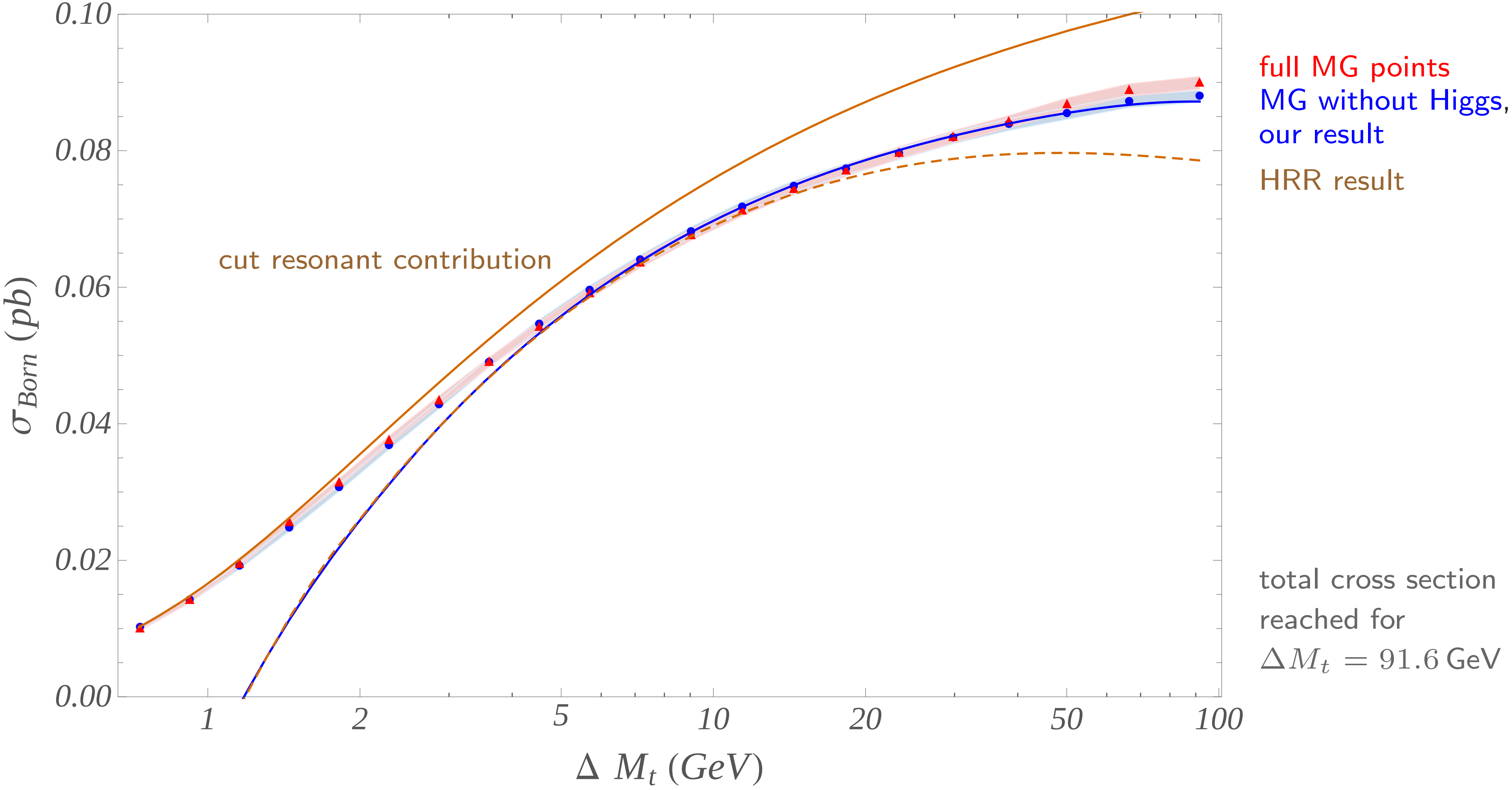
- invariant-mass cuts through "phase-space matching" within non-relativistic EFT (QCD & EW, NLO and some NNLO contributions)
- contributions are expanded for moderate invariant-mass cuts $15\,\text{GeV} \leq \Delta M_t \leq 35\,\text{GeV}$
  $\hookrightarrow$ our result is also valid for larger cuts up to the total cross section.
- EW contributions match leading powers in Λ/m_t of our result
  $\hookrightarrow$ agreement for small cuts ΔM_t

Comparison to MadGraph/MadEvent/MadAnalysis (MG) Alwall et al. '07

$\hookrightarrow$ generated 10^4 events for $e^+e^- \to W^+W^-b\bar{b}$, analyzed cut-dependence

EW tree-level contributions: cut-dependence at threshold

cross section (for $\alpha_s = 0$) at threshold ($s = 4m_t^2$) as a function of the invariant-mass cut ΔM_t



Our result (solid-blue): EW non-resonant NLO + resonant NNLO tree-level contributions

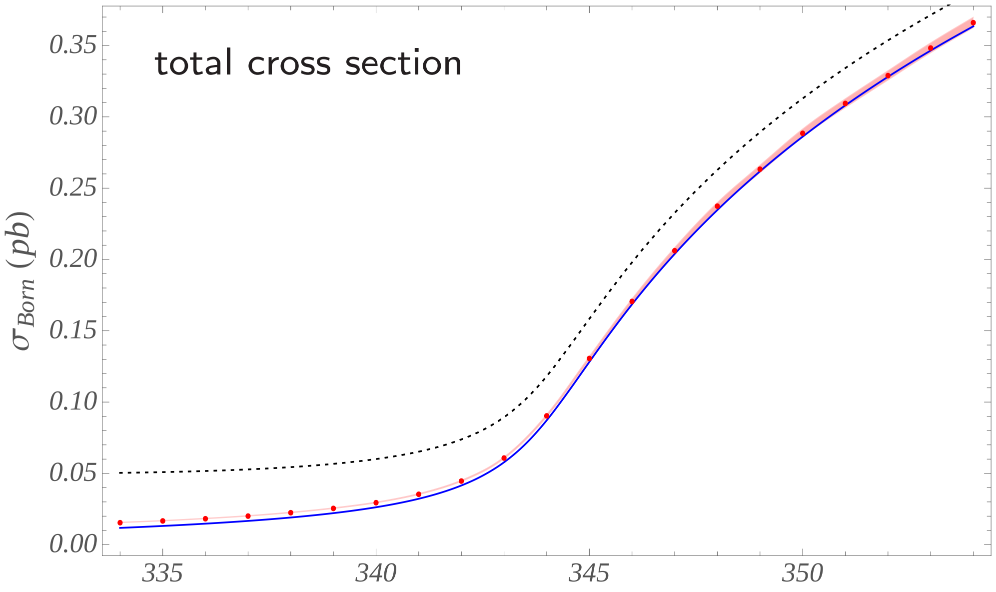
↪ good agreement with MadGraph (MG) for loose cuts $\Delta M_t \gtrsim 5$ GeV

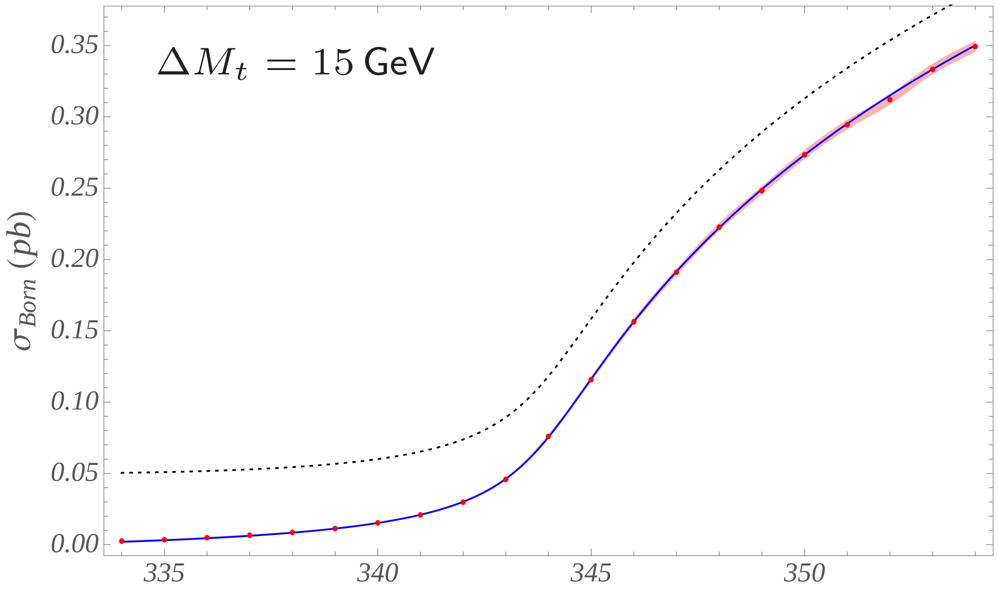
Cut resonant contribution (LO): solid-brown ⇒ good agreement with MG for tight cuts $\Delta M_t \lesssim 1$ GeV

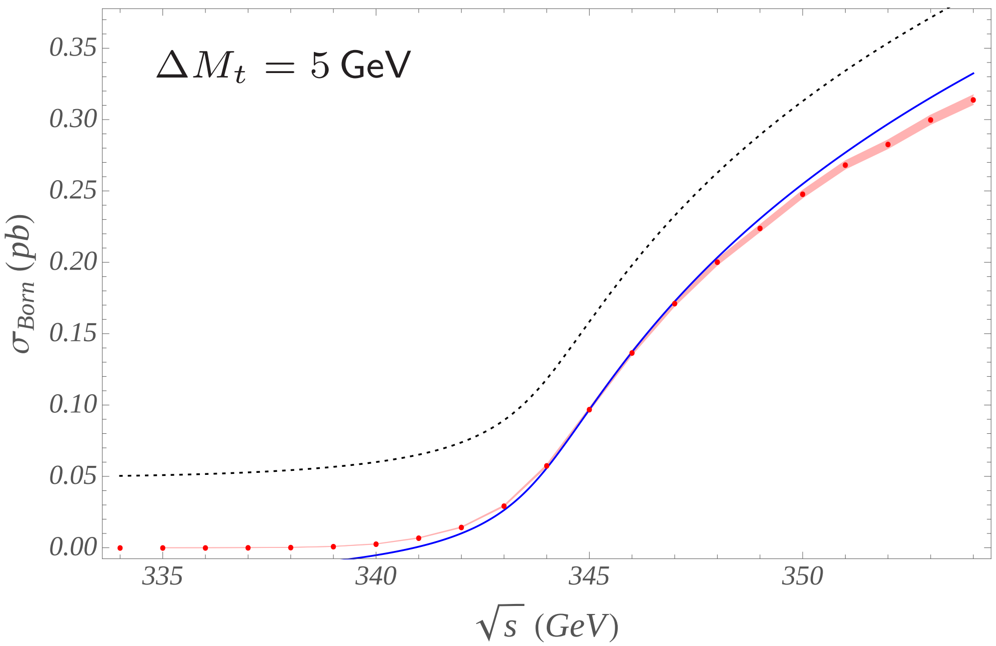
Hoang–Reißer–Ruiz-Femenía (HRR) result: dashed-brown ⇒ agrees with our result for small ΔM_t

EW tree-level contributions: energy-dependence for different cuts

cross section (for $\alpha_s = 0$) as a function of the centre-of-mass energy $\sqrt{s}$







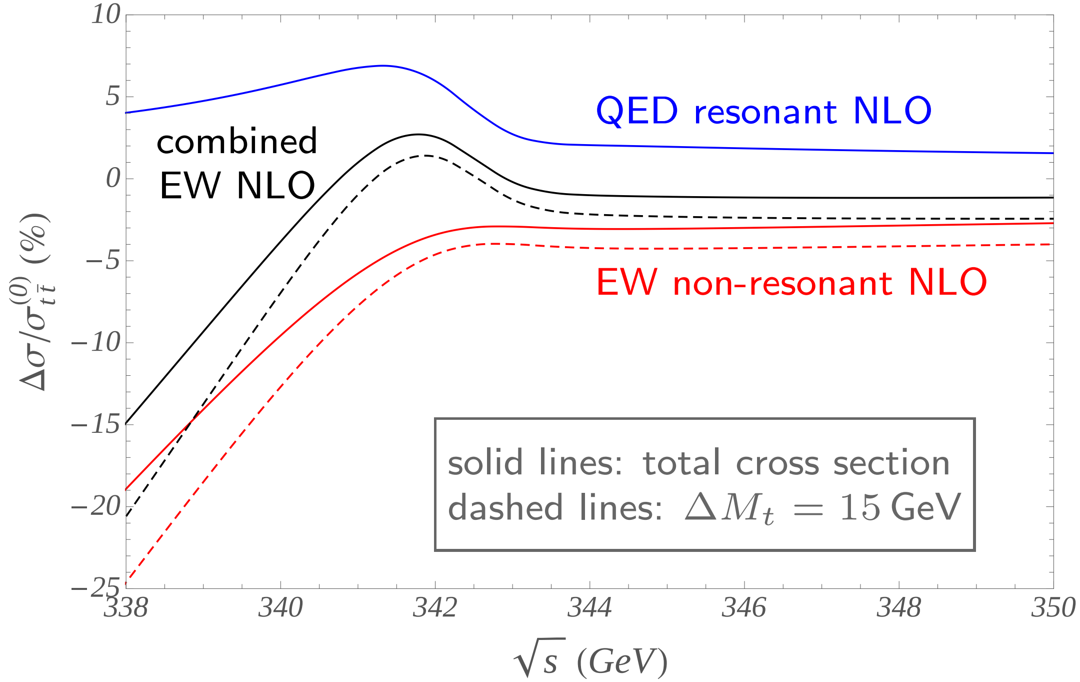
MG (full) points & error band,

EW NNLO tree-level contributions (solid-blue) [resonant + non-resonant],

only resonant contributions (dotted-black)

Full cross section with QCD LO & EW NLO contributions

$[\alpha_s^{\overline{\text{MS}}}(30\,\text{GeV}) = 0.142]$



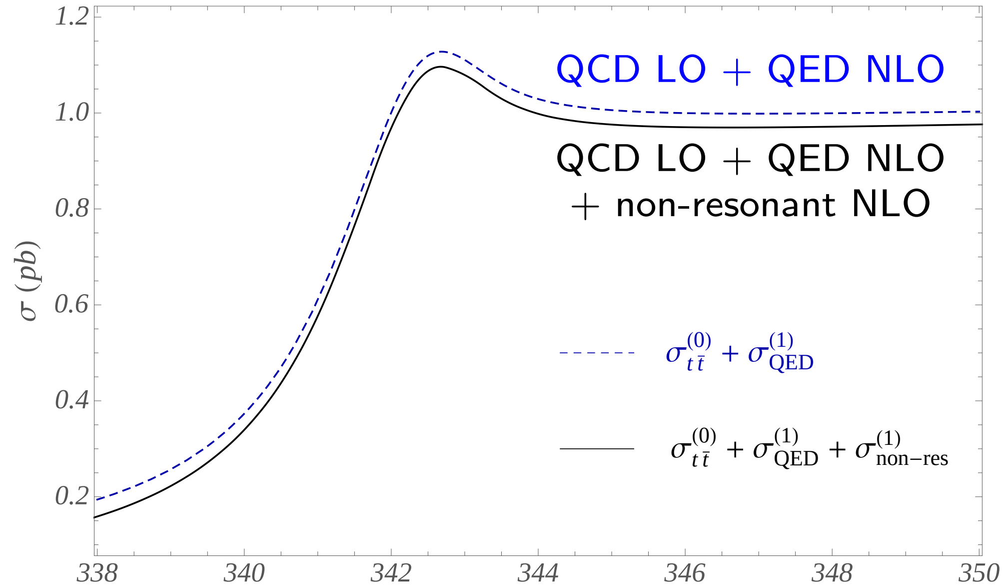
Relative sizes of EW NLO corrections w.r.t. LO (incl. resummed "Coulomb gluons"):

QED resonant correction ("Coulomb photons"),

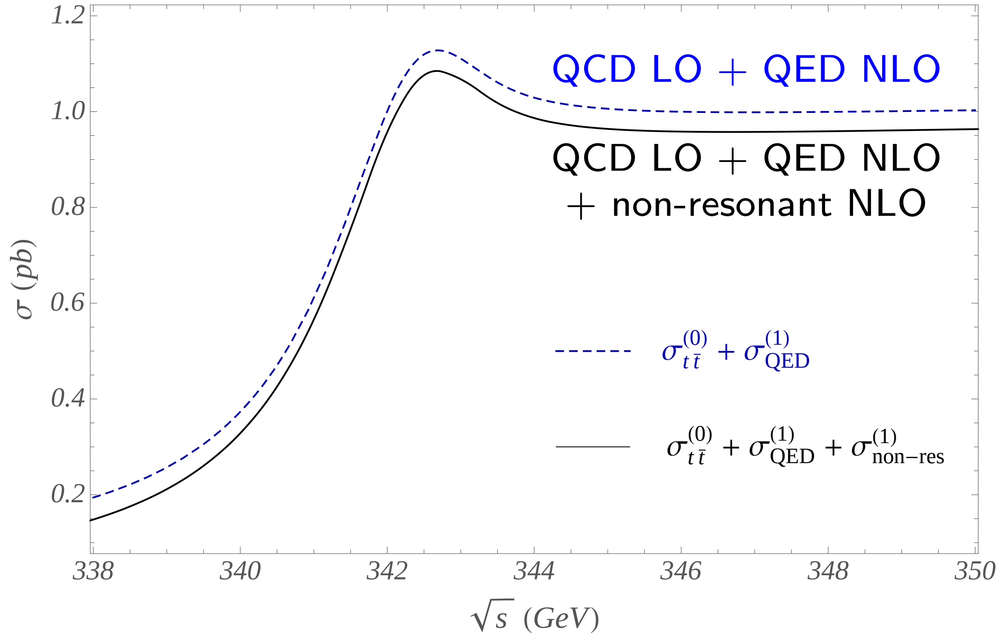
non-resonant NLO correction,

combined EW NLO corrections

Total cross section



Cross section with $\Delta M_t = 15$ GeV



IV Conclusions & outlook

EW non-resonant corrections to $e^+e^- \longrightarrow W^+W^-b\bar{b}$ in the $t\bar{t}$ resonance region

- NLO contribution completed by EW non-resonant contributions for total cross section and with top invariant-mass cuts
- correction of $\sim$ -30 fb (-3% above and much more below threshold) for total cross section, even more with invariant-mass cuts
- good agreement with MadGraph for loose cuts
- good agreement with Hoang–Reißer–Ruiz-Femenía result for small cuts

$\hookrightarrow$ can be added to existing QCD results to improve accuracy of theoretical prediction

Future improvements

- add initial-state radiation and convolution with electron distribution functions
- add gluon exchange to non-resonant contributions $\Rightarrow$ EW NNLO corrections
  $\hookrightarrow$ cancel finite-width divergences $\propto \alpha_s \dfrac{\Gamma_t}{\epsilon}$