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Two-loop electroweak next-to-leading logarithmic corrections to massless fermionic processes

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Overview

I Electroweak corrections at high energies

II 2-loop next-to-leading logarithmic corrections

- extraction of mass-singular logs at 1 and 2 loops
- factorizable and non-factorizable contributions
- treatment of UV singularities

III Results for massless fermionic processes

- calculation of loop integrals
- factorization & exponentiation
- comparison to existing results & applications

IV Summary

I Electroweak (EW) corrections at high energies

EW collider experiments

- today (LEP, Tevatron): energy scales $\lesssim M_{\mathrm{W,Z}}$
- upcoming colliders (LHC, ILC) $\rightarrow$ explore TeV regime
  $\hookrightarrow$ new energy domain $\sqrt{s} \gg M_{\mathrm{W}}$ becomes accessible!

EW radiative corrections at high energies $\sqrt{s} \gg M_{\mathrm{W}}$

$\Rightarrow$ enhanced by large Sudakov logarithms

$$\ln^2\left(\frac{s}{M_{\mathrm{W}}^2}\right) \sim 25 \quad \text{at } \sqrt{s} \sim 1\,\text{TeV}$$

Logs present in exclusive observables with only virtual W and Z bosons (this project), but also in inclusive observables due to Bloch–Nordsieck violations

Ciafaloni, Ciafaloni, Comelli '00,'01; Ciafaloni, Comelli '05

General form of EW corrections for $s \gg M_W^2$

$$\left[L = \ln\left(\frac{s}{M_W^2}\right) \right]$$

1 loop:

$$\alpha \left[C_1^{\mathsf{LL}} L^2 + C_1^{\mathsf{NLL}} L + C_1^{\mathsf{N^2LL}} \right] + \mathcal{O}\left(\frac{M_W^2}{s}\right)$$

$$\downarrow \quad -17\,\% \qquad \downarrow \quad +12\,\% \qquad \downarrow \quad -3\,\%$$

2 loops:

$$\alpha^2 \left[C_2^{\mathsf{LL}} L^4 + C_2^{\mathsf{NLL}} L^3 + C_2^{\mathsf{N^2LL}} L^2 + C_2^{\mathsf{N^3LL}} L + C_2^{\mathsf{N^4LL}} \right] + \mathcal{O}\left(\frac{M_W^2}{s}\right)$$

$$\downarrow \quad +1.7\,\% \qquad \downarrow \quad -1.8\,\% \qquad \downarrow \quad +1.2\,\% \qquad \downarrow \quad -0.3\,\%$$

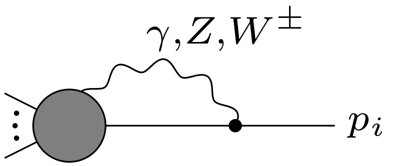
[percentages for $\sigma(u\bar{u} \to d\bar{d})$ at $\sqrt{s} = 1\,\text{TeV}$
B.J., Kühn, Penin, Smirnov '05]

Theoretical prediction with accuracy $\sim 1\,\%$ required

⇒ 2-loop corrections important

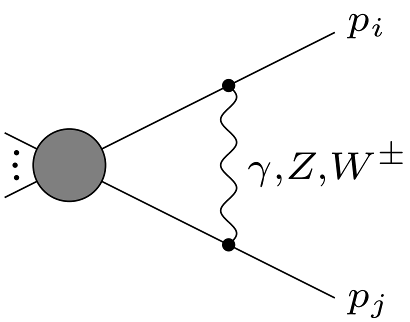
⇒ 2-loop LL approximation not sufficient

Origin of logarithms $\ln(s/M_W^2)$ in virtual corrections



mass-singularities from virtual gauge bosons $(\gamma, Z, W^{\pm})$
coupling to on-shell external leg

$\longrightarrow$ single logs from collinear region



special case:
gauge bosons exchanged between **2** on-shell external legs
$\longrightarrow$ double logs from soft–collinear region

For massless photons: $\log \rightsquigarrow \frac{1}{\epsilon}$ in $D = 4 - 2\epsilon$ dimensions
$\Rightarrow$ count $1/\epsilon$ poles like logs for logarithmic approximations (LL, NLL, ...)

EW 1-loop LLs & NLLs for arbitrary processes are universal
$\hookrightarrow$ depend only on quantum numbers of external particles — Denner, Pozzorini '00, '01

Approaches for virtual 2-loop EW corrections at high energies

Resummation of 1-loop result to all orders:

- LL for arbitrary processes — Fadin, Lipatov, Martin, Melles '99
- NLL for arbitrary processes ($M_\mathrm{Z} = M_\mathrm{W}$) — Melles '00, '01
- N^{2}LL for massless $f\bar{f} \to f'\bar{f}'$ ($M_\mathrm{Z} = M_\mathrm{W}$) — Kühn, Penin, Smirnov '99, '00; Kühn, Moch, Penin, Smirnov '01

$\longrightarrow$ apply evolution equations to spontaneously broken SU(2)×U(1) EW model
$\hookrightarrow$ rely on splitting of EW theory into symmetric SU(2)×U(1) and QED regime

Diagrammatic 2-loop calculations to check & extend resummation predictions:

- LL for fermionic form factor — Melles '00; Hori, Kawamura, Kodaira '00
- LL for arbitrary processes — Beenakker, Werthenbach '00, '01
- angular-dependent NLLs for arbitrary processes — Denner, Melles, Pozzorini '03
- complete NLL for fermionic form factor — Pozzorini '04
- N^{3}LL for fermionic form factor ($M_\mathrm{Z} = M_\mathrm{W}$)
  $\hookrightarrow$ N^{3}LL for massless $f\bar{f} \to f'\bar{f}'$ ($M_\mathrm{Z} \approx M_\mathrm{W}$) via evolution equations
  B.J., Kühn, Moch '03; B.J., Kühn, Penin, Smirnov '04, '05

II 2-loop next-to-leading logarithmic corrections

Goal: virtual 2-loop EW corrections for arbitrary processes in NLL accuracy

$\hookrightarrow$ further check of evolution equation predictions

$\hookrightarrow$ provide better accuracy for arbitrary $2 \to n$ processes

Implement

- different large kinematical invariants $|(p_i + p_j)^2| \sim Q^2 \gg M_\mathrm{W}^2$
- different heavy particle masses $M_\mathrm{W}^2 \sim M_\mathrm{Z}^2 \sim m_\mathsf{top}^2 \sim M_\mathsf{Higgs}^2$ (light masses = 0)

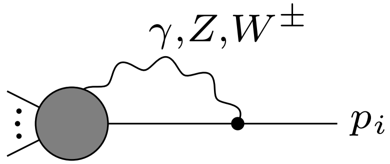
$\Rightarrow$ Logs $L = \ln\left(\frac{Q^2}{M_\mathrm{W}^2}\right)$ and $\frac{1}{\epsilon}$ poles (from virtual photons)

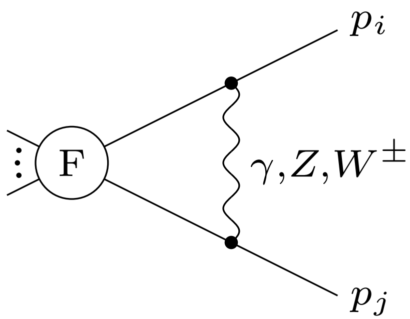
1 loop: LL $\to \epsilon^{-2},\ L\epsilon^{-1},\ L^2,\ L^3\epsilon,\ L^4\epsilon^2$; NLL $\to \epsilon^{-1},\ L,\ L^2\epsilon,\ L^3\epsilon^2$

2 loops: LL $\to \epsilon^{-4},\ L\epsilon^{-3},\ L^2\epsilon^{-2},\ L^3\epsilon^{-1},\ L^4$; NLL $\to \epsilon^{-3},\ L\epsilon^{-2},\ L^2\epsilon^{-1},\ L^3$

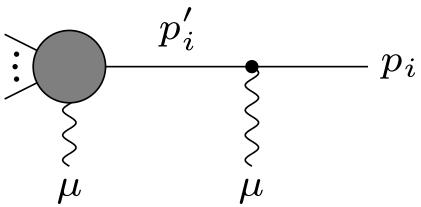
$\Rightarrow$ NLL coefficients involve small logs $\ln\left(\frac{|(p_i + p_j)^2|}{Q^2}\right)$ and $\ln\left(\frac{M_\mathrm{Z}^2, m_\mathsf{top}^2, M_\mathsf{Higgs}^2}{M_\mathrm{W}^2}\right)$

Extraction of NLL mass singularities at 1 loop

Contributions originate from  in the collinear region

Isolate factorizable contributions: 

- gauge boson momentum set to zero in tree subdiagram (F)
- soft–collinear approximation for loop vertices combined with propagators:



$i\not{p}_i' \cdot i\gamma^\mu \to -2{p_i'}^\mu$ (for massless external fermions),
extension of *eikonal approximation*,
valid for soft and/or collinear gauge boson momenta

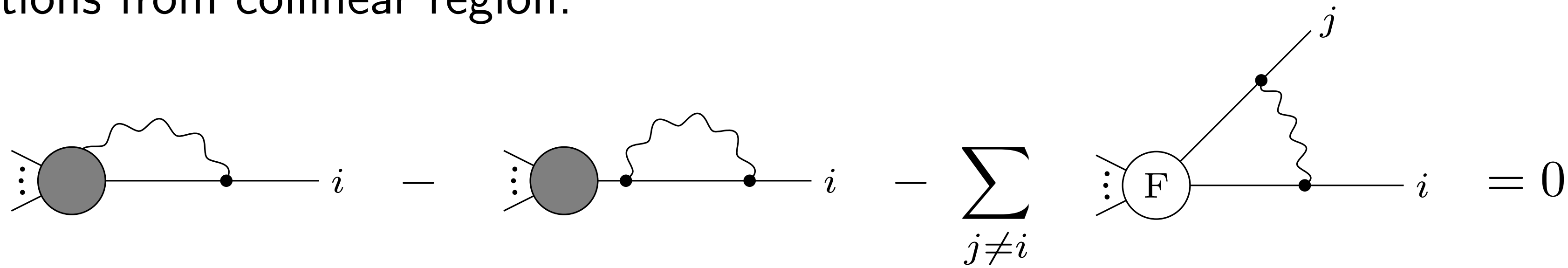
↪ eliminates Dirac structure of loop corrections

⇒ loop integrals independent of structure of Born matrix element

The factorizable contributions contain all soft and/or collinear NLL mass singularities.

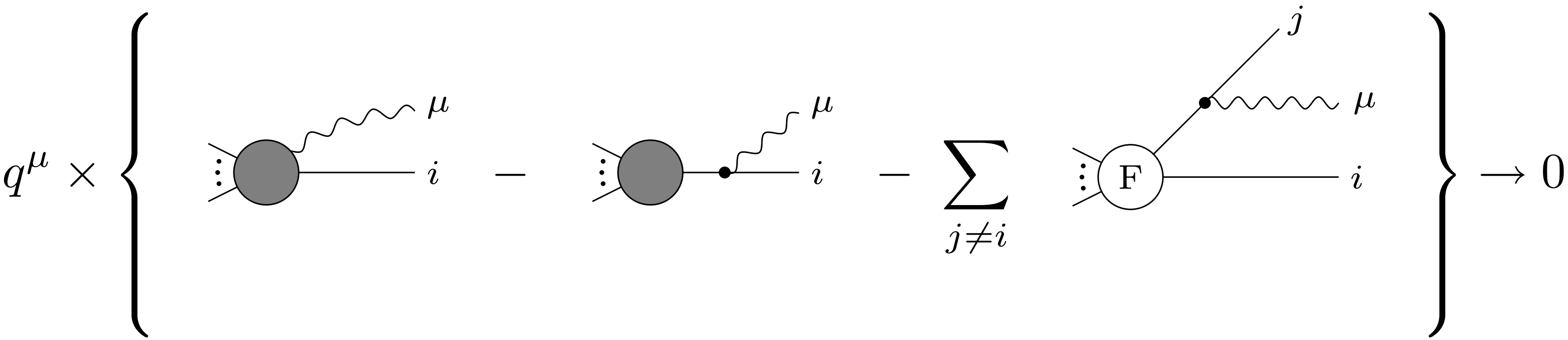
Remaining non-factorizable contributions

Contributions from collinear region:



Cancellation mechanism:

- collinear vertex $\propto$ gauge boson momentum q^μ
- collinear Ward identities for EW theory: Denner, Pozzorini '00, '01

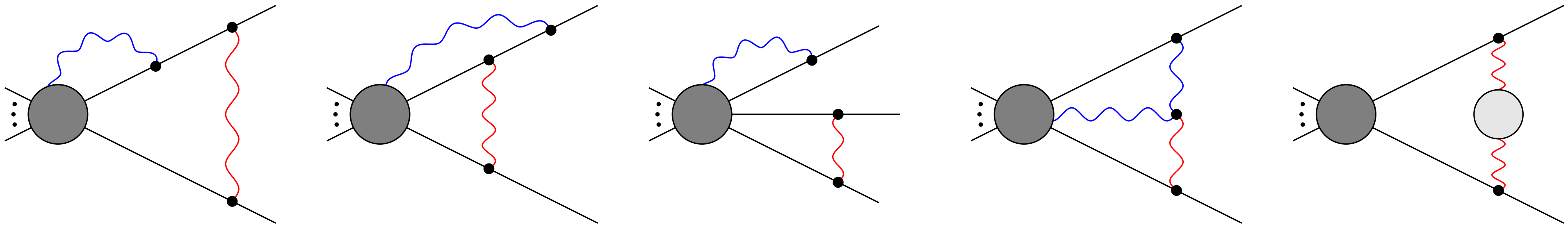


in the collinear limit

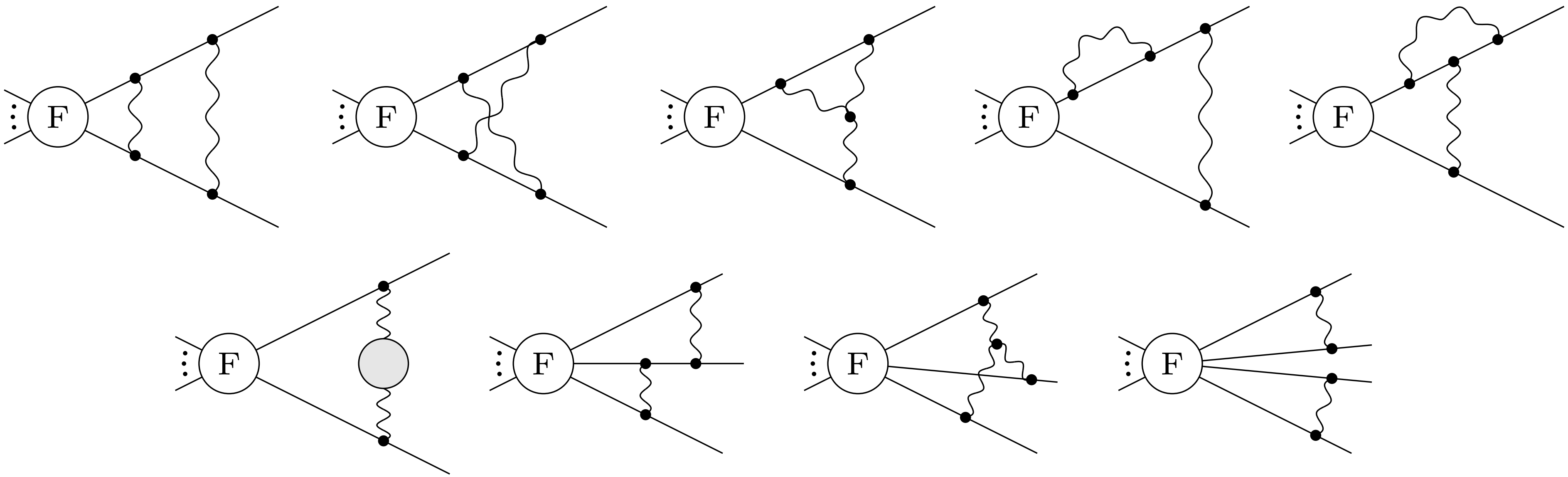
$\Rightarrow$ only calculation of factorizable contributions needed

Extraction of NLL mass singularities at 2 loops

$\hookrightarrow$ soft×soft and soft×collinear contributions:



Factorizable contributions:



- calculated with soft–collinear approximation and projection techniques
- remaining non-factorizable contributions vanish due to collinear Ward identities

Denner, B.J., Pozzorini '06

Treatment of ultraviolet (UV) singularities & renormalization

UV $1/\epsilon$ poles in subdiagrams with scale μ^2_{loop}
and renormalization at scale μ^2_{R} $\Big\}$ $\Rightarrow$ logs $\ln\left(\frac{\mu^2_{\text{R}}}{\mu^2_{\text{loop}}}\right)$ $\Rightarrow$ possibly NLL

UV subtraction: remove $1/\epsilon$ poles from UV-singular subdiagrams and counterterms

Advantages:

- UV NLL contributions from hard subdiagrams completely shifted to counterterms
  $\hookrightarrow$ no need to calculate UV-singular loops in internal parts of tree subdiagrams
- can use soft–collinear approximation (not valid in UV regime!) also for hard subdiagrams which produce UV NLL contributions

Renormalization:

- use couplings in Born matrix elements renormalized at $\mu^2_{\text{R}} = Q^2$
  $\hookrightarrow$ no counterterm contributions from Born amplitude
  $\hookrightarrow$ loop corrections universal & independent of inner structure of Born amplitude
  $\hookrightarrow$ renormalization scale can be changed for specific matrix elements
- renormalization of couplings in loops at arbitrary scale μ^2_{R}

III Results for massless fermionic processes

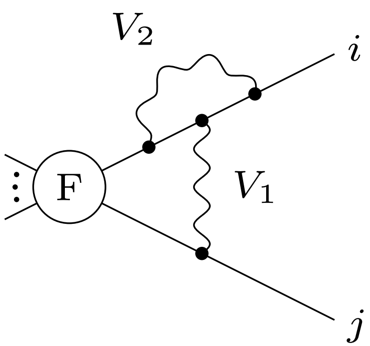
Evaluation of factorizable contributions

Calculate & check loop integrals with 2 independent methods:

- automatized algorithm based on sector decomposition — Denner, Pozzorini '04
- combination of expansion by regions & Mellin–Barnes representations — Smirnov, B.J. '06 & refs. therein

↪ extended for different hard scales $(p_i + p_j)^2 \sim Q^2$ and soft scales $M_i^2 \sim M_\mathrm{W}^2$

Example:



$$= \underset{\text{Born amplitude factorized}}{\mathcal{M}_0} \underbrace{\sum_{V_1, V_2 = \gamma, Z, W^\pm}}_{\text{sum over various gauge bosons}} \overbrace{I_i^{V_2} I_i^{V_1} I_i^{\bar{V}_2} I_j^{\bar{V}_1}}^{\text{isospin matrices @ external legs}} \underbrace{D(M_{V_1}, M_{V_2}; p_i \cdot p_j)}_{\text{scalar 2-loop integral}}$$

$+$ sum over external legs i, j

NLL result for massless fermionic processes $f_1 f_2 \longrightarrow f_3 \cdots f_n$ up to 2 loops

$$\boxed{\mathcal{M} = \mathcal{M}_0 \, F^{\mathsf{sew}} \, F^{\mathrm{Z}} \, F^{\mathsf{em}}}$$

symmetric-electroweak factor: $$F^{\mathsf{sew}} = \exp\left[\frac{\alpha}{4\pi} F_1^{\mathsf{sew}} + \left(\frac{\alpha}{4\pi}\right)^2 G_2^{\mathsf{sew}}\right]$$

electromagnetic factor: $$F^{\mathsf{em}} = \exp\left[\frac{\alpha}{4\pi} \Delta F_1^{\mathsf{em}} + \left(\frac{\alpha}{4\pi}\right)^2 \Delta G_2^{\mathsf{em}}\right]$$

terms from $M_{\mathrm{Z}} \neq M_{\mathrm{W}}$: $$F^{\mathrm{Z}} = 1 + \frac{\alpha}{4\pi} \Delta F_1^{\mathrm{Z}}$$

- F^{sew} equals result from symmetric SU(2)×U(1) theory with $M_\gamma = M_{\mathrm{W}} = M_{\mathrm{Z}}$
- exponentiation of 1-loop terms F_1^{sew} and ΔF_1^{em} (found from fixed-order calculation!)
- electromagnetic terms in F^{em} factorize and exponentiate separately
  $\hookrightarrow$ separation of photonic singularities possible
- loop correction factors F^{sew}, F^{em} and F^{Z} are universal,
  $\hookrightarrow$ depend only on quantum numbers of external particles

Exponentiated 1-loop terms: LLs & NLLs

$$F_1^{\text{sew}} = -\frac{1}{2}\left(L^2 + \frac{2}{3}L^3\epsilon + \frac{1}{4}L^4\epsilon^2 - 3L - \frac{3}{2}L^2\epsilon - \frac{1}{2}L^3\epsilon^2\right)\sum_{i=1}^{n}\left(\frac{Y_i^2}{4c_{\text{w}}^2} + \frac{C_i}{s_{\text{w}}^2}\right)$$
$$+\left(L + \frac{1}{2}L^2\epsilon + \frac{1}{6}L^3\epsilon^2\right)\sum_{i=1}^{n}\sum_{\substack{j=1\\ j\neq i}}^{n}\ln\left(\frac{-(p_i+p_j)^2}{Q^2}\right)\sum_{V=\gamma,Z,W^\pm} I_i^{\bar{V}} I_j^{V}$$

$$\Delta F_1^{\text{em}} = -\frac{1}{2}\left(2\epsilon^{-2} - L^2 - \frac{2}{3}L^3\epsilon - \frac{1}{4}L^4\epsilon^2 + 3\epsilon^{-1} + 3L + \frac{3}{2}L^2\epsilon + \frac{1}{2}L^3\epsilon^2\right)\sum_{i=1}^{n} Q_i^2$$
$$-\left(\epsilon^{-1} + L + \frac{1}{2}L^2\epsilon + \frac{1}{6}L^3\epsilon^2\right)\sum_{i=1}^{n}\sum_{\substack{j=1\\ j\neq i}}^{n}\ln\left(\frac{-(p_i+p_j)^2}{Q^2}\right) Q_i Q_j$$

$$\Delta F_1^{\text{Z}} = \left(L + L^2\epsilon + \frac{1}{2}L^3\epsilon^2\right)\ln\left(\frac{M_{\text{Z}}^2}{M_{\text{W}}^2}\right)\sum_{i=1}^{n}\left(\frac{c_{\text{w}}}{s_{\text{w}}}T_i^3 - \frac{s_{\text{w}}}{c_{\text{w}}}\frac{Y_i}{2}\right)^2$$

Additional 2-loop terms with 1-loop β-function coefficients: only NLLs

$$G_2^{\text{sew}} = \frac{1}{6}L^3\sum_{i=1}^{n}\left(b_1^{(1)}\frac{Y_i^2}{4c_{\text{w}}^2} + b_2^{(1)}\frac{C_i}{s_{\text{w}}^2}\right)$$

$$\Delta G_2^{\text{em}} = \left(\frac{3}{4}\epsilon^{-3} + L\epsilon^{-2} + \frac{1}{2}L^2\epsilon^{-1}\right) b_{\text{QED}}^{(1)}\sum_{i=1}^{n} Q_i^2$$

$[\mu_{\text{R}}^2 = M_{\text{W}}^2]$

Comparison to existing results

- previous results for form factor and angular-dependent NLLs reproduced and extended — Denner, Melles, Pozzorini '03; Pozzorini '04
- structure of symmetric-electroweak NLL corrections in complete analogy with Catani's formula for massless QCD — Catani '98
- agreement with general resummation predictions based on evolution equations — Melles '00, '01

Application to massless 4-fermion scattering

- neutral current $f\bar{f} \to f'\bar{f}'$: agreement (NLL), — B.J., Kühn, Penin, Smirnov '05
  additional contributions $\propto s_{\mathrm{w}}^2 \ln(M_{\mathrm{Z}}^2/M_{\mathrm{W}}^2)$
- charged current $f_1\bar{f}_2 \to f_3\bar{f}_4$: new NLL result

IV Summary

Massless fermionic processes $f_1 f_2 \to f_3 \cdots f_n$

with different $|(p_i + p_j)^2| \gg M_W^2$ and different masses $M_W^2 \sim M_Z^2 \sim m_{top}^2 \sim M_{Higgs}^2$:

- complete EW NLL corrections in $D = 4 - 2\epsilon$ dimensions
  $\hookrightarrow$ Denner, B.J., Pozzorini, **hep-ph/0608326**
- factorizable contributions calculated with 2 independent methods:
  sector decomposition; expansion by regions & Mellin–Barnes
- non-factorizable contributions shown to vanish due to collinear Ward identities
- result expressed by exponentiated 1-loop terms and β-function coefficients
- agreement with existing results

Towards EW NLL corrections for arbitrary processes

- advantage: large parts of method are general
- extension to massive fermionic & arbitrary processes
  $\hookrightarrow$ work in progress . . .