Pheno 2009 Symposium
University of Wisconsin at Madison
May 11–13, 2009

Process-independent determination of two-loop electroweak logarithms

Bernd Jantzen
RWTH Aachen University

In collaboration with **Ansgar Denner** and **Stefano Pozzorini**

I	Electroweak (EW) corrections at high energies
II	Two-loop next-to-leading logarithmic (NLL) corrections
III	Results for processes involving massless and massive fermions
IV	Summary & outlook

Nucl. Phys. B761 (2007) 1–62 [arXiv:hep-ph/0608326] (massless fermions)
JHEP 11 (2008) 062 [arXiv:0809.0800 [hep-ph]] (heavy quarks)

I Electroweak corrections at high energies

Precision collider physics

Precision measurements at colliders and theoretical predictions with electroweak (EW) + QCD corrections enable us to test the Standard Model at various energy scales:

- up to now (LEP, Tevatron) at energy scales $\lesssim M_{\mathrm{W,Z}}$
- upcoming colliders (LHC, ILC/CLIC) → reach TeV regime

  ↪ new energy domain $\sqrt{s} \gg M_{\mathrm{W}}$ becomes accessible!

EW radiative corrections at high energies $\sqrt{s} \gg M_{\mathrm{W}}$

⇒ enhanced by large Sudakov logarithms

$$\text{per loop:} \quad \ln^2\left(\frac{s}{M_{\mathrm{W}}^2}\right) \sim 25 \quad \text{at } \sqrt{s} \sim 1\,\text{TeV}$$

↪ corrections rise with energy

Origin of large EW logs

- mass singularities: real or virtual emission of soft/collinear gauge bosons from external particles
- remnants from UV singularities

Massless gauge bosons

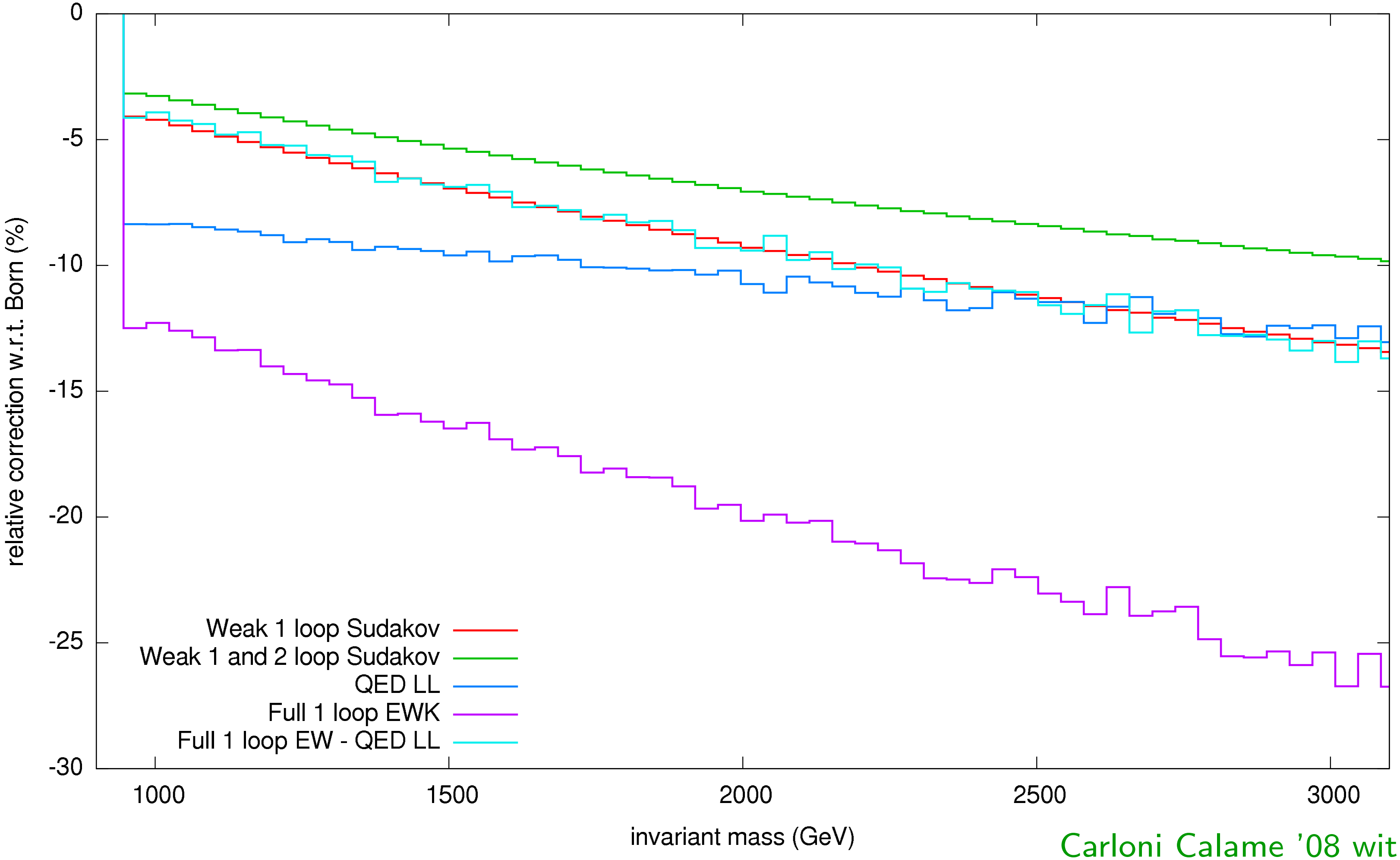
real emission of soft/collinear photons/gluons cannot be detected separately
↪ mass singularities cancel between real & virtual corrections (KLN theorem)

Massive gauge bosons

real emission of W's, Z's can (in principle) be detected separately
↪ only virtual corrections: large logs remain present in exclusive observables,
↪ even in inclusive observables (Bloch–Nordsieck violations)

EW 1-/2-loop corrections at the LHC

Drell–Yan $pp \to \mu^+\mu^-$: (electro)weak 1-loop & 2-loop corrections



Carloni Calame ’08 with HORACE
and logarithmic (“Sudakov”) results from
B.J., Kühn, Penin, Smirnov ’05

$\Rightarrow$ logarithmic approximation very good at high energies

$\Rightarrow$ 2-loop effects $\sim \mathcal{O}(\%)$

cf. Les Houches 2007 report, arXiv:0803.0678 [hep-ph]

Existing virtual EW 2-loop corrections

LL = leading logarithmic ($\ln^4$ at 2 loops), NLL = next-to-leading-logarithmic ($\ln^3$), …

Resummation of 1-loop results using evolution equations:

$$\alpha^2 \left[\underbrace{\underbrace{C_{\mathrm{LL}} \ln^4\left(\frac{s}{M_{\mathrm{W}}^2}\right)}_{\text{Fadin, Lipatov, Martin, Melles '99}} + \underbrace{C_{\mathrm{NLL}} \ln^3\left(\frac{s}{M_{\mathrm{W}}^2}\right)}_{\text{Melles '00, '01}}}_{\text{arbitrary processes}} + \underbrace{\underbrace{C_{\mathrm{N^2LL}} \ln^2\left(\frac{s}{M_{\mathrm{W}}^2}\right)}_{\text{Kühn, Moch, Penin, Smirnov '99–'01}} + \underbrace{C_{\mathrm{N^3LL}} \ln\left(\frac{s}{M_{\mathrm{W}}^2}\right)}_{\text{B.J., Kühn, Moch, Penin, Smirnov '03–'05}}}_{\text{massless } f\,\bar{f} \to f'\,\bar{f}' \text{ processes}} \right]$$

+ SCET approach [Chiu, Golf, Kelley, Manohar '07, '08]
+ N^{2}LL for $e^+ e^- \to W^+ W^-$ [Kühn, Metzler, Penin '07]

Explicit 2-loop calculations based on spontaneously broken SU(2)×U(1), $M_{\mathrm{Z}} \neq M_{\mathrm{W}}$:

$$\alpha^2 \left[\underbrace{\underbrace{C_{\mathrm{LL}} \ln^4\left(\frac{s}{M_{\mathrm{W}}^2}\right)}_{\text{Melles '00; Hori, Kawamura, Kodaira '00; Beenakker, Werthenbach '00, '01}} + \underbrace{C_{\mathrm{NLL}}^{\mathrm{ang}} \ln\left(\frac{t}{s}\right) \ln^3\left(\frac{s}{M_{\mathrm{W}}^2}\right)}_{\text{Denner, Melles, Pozzorini '03}}}_{\text{arbitrary processes}} + \underbrace{\underbrace{C_{\mathrm{NLL}}^{\mathrm{rem}} \ln^3\left(\frac{s}{M_{\mathrm{W}}^2}\right)}_{\text{Pozzorini '04; Denner, B.J., Pozzorini '06, '08}}}_{n\text{-fermion processes}} \right]$$

II Two-loop next-to-leading logarithmic corrections

Goal: virtual 2-loop EW corrections for arbitrary processes in NLL accuracy

Process-independent: separate loop integrals from Born matrix elements

$\hookrightarrow$ already completed: processes involving massless & massive external fermions

Parameters: $[D = 4 - 2\epsilon]$

- different large kinematical invariants $r_{ij} = (p_i + p_j)^2 \sim Q^2 \gg M_\mathrm{W}^2$
- different heavy particle masses $M_\mathrm{W}^2 \sim M_\mathrm{Z}^2 \sim m_\mathrm{t}^2 \sim M_\mathsf{Higgs}^2$
- massive top quark, other fermions massless

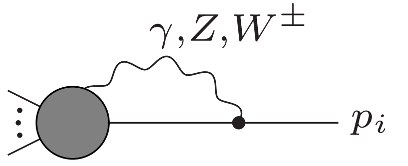
$\Rightarrow$ logs $L = \ln\left(\dfrac{Q^2}{M_\mathrm{W}^2}\right)$ and $\dfrac{1}{\epsilon}$ poles (from massless photons, counted like logs)

1 loop: LL $\rightarrow \epsilon^{-2},\ L\epsilon^{-1},\ L^2,\ L^3\epsilon,\ L^4\epsilon^2;$ NLL $\rightarrow \epsilon^{-1},\ L,\ L^2\epsilon,\ L^3\epsilon^2$

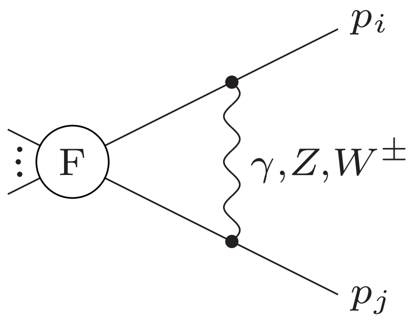
2 loops: LL $\rightarrow \epsilon^{-4},\ L\epsilon^{-3},\ L^2\epsilon^{-2},\ L^3\epsilon^{-1},\ L^4;$ NLL $\rightarrow \epsilon^{-3},\ L\epsilon^{-2},\ L^2\epsilon^{-1},\ L^3$

$\Rightarrow$ NLL coefficients involve small logs $\ln\left(\dfrac{-r_{ij}}{Q^2}\right)$ and $\ln\left(\dfrac{M_\mathrm{Z}^2, m_\mathrm{t}^2}{M_\mathrm{W}^2}\right)$

Extraction of NLL contributions



Logs originate from mass singularities when a virtual gauge boson ($\gamma, Z, W^\pm$) couples to an on-shell external leg
→ single log from collinear region (+ UV logs)

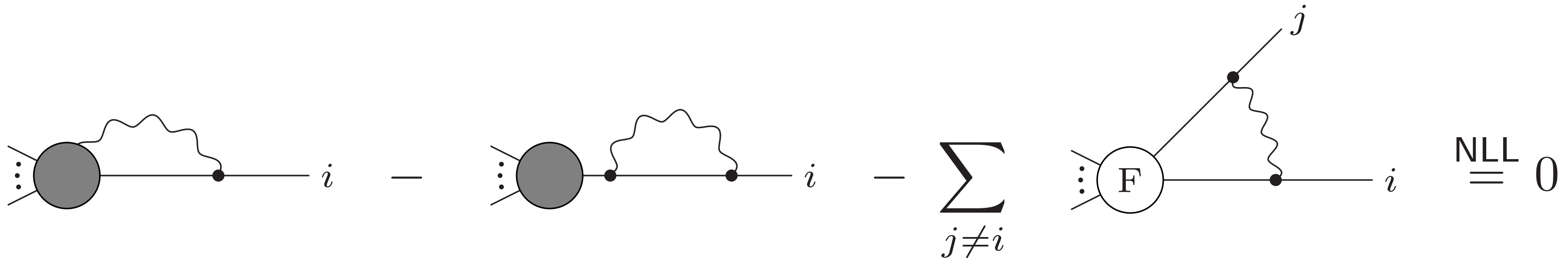


Isolate factorizable contributions:
gauge boson exchanged between external legs;

separate loop integral from Born diagram Ⓕ
via soft–collinear approximation
→ double log from soft & collinear region

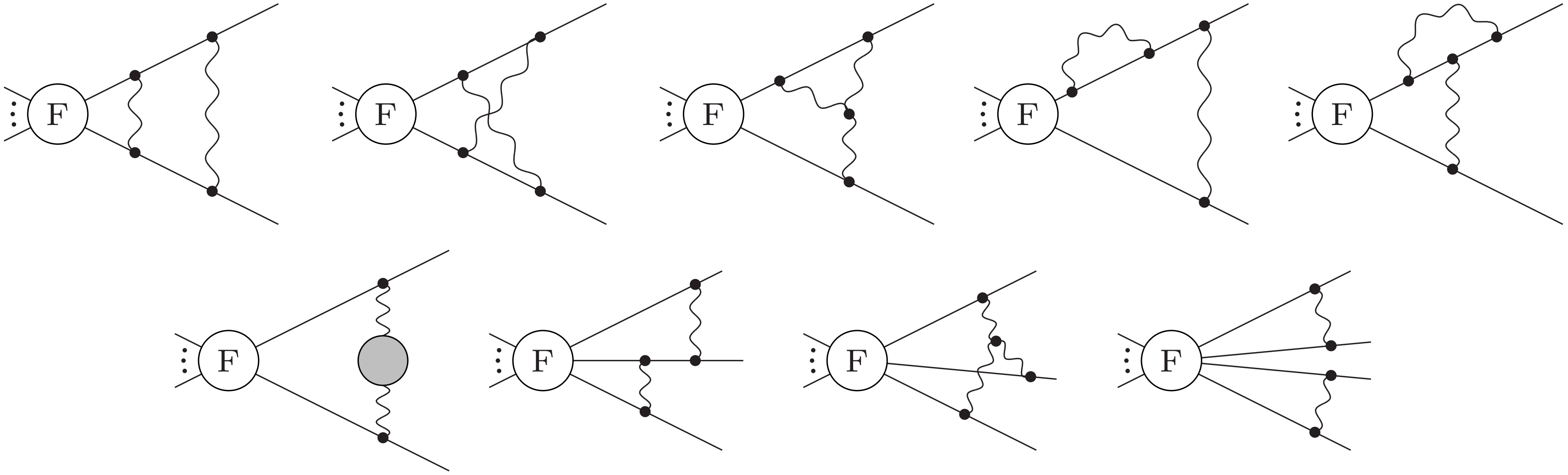
Remaining non-factorizable contributions: collinear Ward identities

Denner, Pozzorini '00, '01



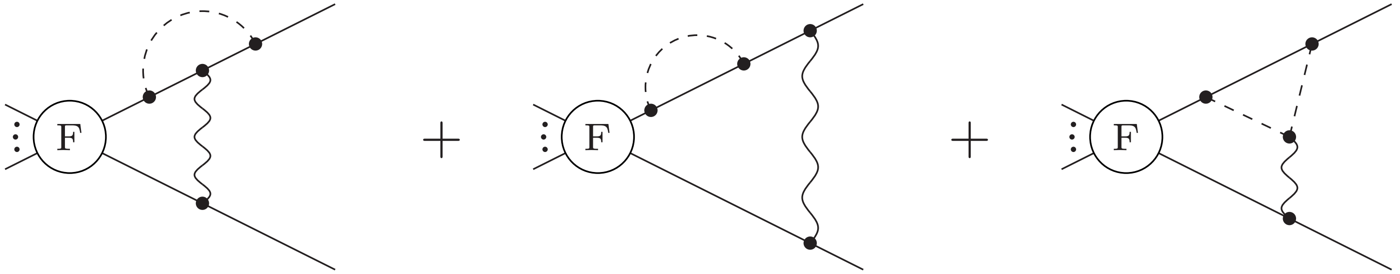
The factorizable contributions contain all soft or collinear NLL mass singularities.

Factorizable contributions at 2 loops:



↪ non-factorizable contributions vanish

Yukawa couplings of massive fermions to Higgs & Goldstone bosons
↪ only three non-suppressed factorizable diagrams:



$$\overset{\text{NLL}}{=} 0$$

⇒ Sum vanishes due to gauge invariance of Yukawa interaction

⇝ Yukawa interaction contributes only to wave-function renormalization

III Results for processes involving massless and massive fermions

Evaluation of factorizable contributions

Loop integrals calculated with two independent methods:

- automatized algorithm based on sector decomposition Denner, Pozzorini '04
- combination of expansion by regions & Mellin–Barnes representations

B.J., Smirnov '06 & refs. therein

All relevant combinations of $\left\{\begin{matrix}\text{massless}\\\text{massive}\end{matrix}\right\}\left\{\begin{matrix}\text{external}\\\text{internal}\end{matrix}\right\}$ fermions evaluated explicitly!

Result for amplitude of fermionic processes $f_1 f_2 \to f_3 \cdots f_n$ in $\mathcal{O}(\alpha^2)$

$$\mathcal{M} \overset{\text{NLL}}{=} \underbrace{\exp\big(\Delta F^{\text{em}}\big)}_{\substack{\text{electromagnetic}\\ M_\gamma = 0}} \times \underbrace{\exp\big(F^{\text{sew}}\big)}_{\substack{\text{symmetric-electroweak}\\ M_\gamma = M_\text{Z} = M_\text{W}}} \times \underbrace{\big(1 + \Delta F^{\text{Z}}\big)}_{\substack{\text{corrections}\\ \text{from } M_\text{Z} \neq M_\text{W}}} \times \underbrace{\mathcal{M}_0}_{\text{Born}}$$

- universal result: F^{sew}, ΔF^{em}, ΔF^{Z} depend only on external quantum numbers
- electromagnetic singularities (in ΔF^{em}) factorized → separable

$$\mathcal{M} \overset{\text{NLL}}{=} \exp\big(\Delta F^{\text{em}}\big) \times \exp\big(F^{\text{sew}}\big) \times \big(1+\Delta F^{\text{Z}}\big) \times \mathcal{M}_0$$

Symmetric-electroweak terms: independent of fermion masses

$$\begin{aligned} F^{\text{sew}} = \frac{1}{2}\sum_{i=1}^{n}\Bigg\{ & -\frac{\alpha}{4\pi}\Bigg[\sum_{j\neq i}\sum_{V=\gamma,Z,W^\pm}\overbrace{I_i^{\bar{V}} I_j^{V}}^{\text{isospin matrices applied to external legs}} I_{ij}(\epsilon, M_{\text{W}}) + \overbrace{\frac{z_i^{\text{Yuk}}\, m_{\text{t}}^2}{4 s_{\text{w}}^2 M_{\text{W}}^2}\Big(L+\frac{1}{2}L^2\epsilon+\frac{1}{6}L^3\epsilon^2+\mathcal{O}(\epsilon^3)\Big)}^{\text{Yukawa contribution}}\Bigg] \\ & + \Big(\frac{\alpha}{4\pi}\Big)^2\Bigg[\frac{b_1^{(1)}}{c_{\text{w}}^2}\Big(\frac{Y_i}{2}\Big)^2 + \frac{b_2^{(1)}}{s_{\text{w}}^2}C_i^{\text{w}}\Bigg] J_{ii}(\epsilon, M_{\text{W}}, \mu_{\text{R}}^2)\Bigg\}, \end{aligned}$$

$$I_{ij}(\epsilon, M_{\text{W}}) \overset{\text{NLL}}{=} -L^2 - \frac{2}{3}L^3\epsilon - \frac{1}{4}L^4\epsilon^2 + \Big[3 - 2\ln\Big(\frac{-r_{ij}}{Q^2}\Big)\Big]\Big(L+\frac{1}{2}L^2\epsilon+\frac{1}{6}L^3\epsilon^2\Big) + \mathcal{O}(\epsilon^3),$$

$$J_{ij}(\epsilon, M_{\text{W}}, \mu^2) = \frac{1}{\epsilon}\Big[I_{ij}(2\epsilon, M_{\text{W}}) - \Big(\frac{Q^2}{\mu^2}\Big)^{\epsilon} I_{ij}(\epsilon, M_{\text{W}})\Big]$$

Terms from $M_{\text{Z}} \neq M_{\text{W}}$:

$$\Delta F^{\text{Z}} \overset{\text{NLL}}{=} \frac{1}{2}\sum_{i=1}^{n}\frac{\alpha}{4\pi}\big(I_i^{\text{Z}}\big)^2 \overbrace{\ln\Big(\frac{M_{\text{Z}}^2}{M_{\text{W}}^2}\Big)\big(2L+2L^2\epsilon+L^3\epsilon^2\big)}^{=I_{ii}(\epsilon,M_{\text{Z}})-I_{ii}(\epsilon,M_{\text{W}})} + \mathcal{O}(\epsilon^3)$$

$$\mathcal{M} \overset{\text{NLL}}{=} \exp\big(\Delta F^{\text{em}}\big) \times \exp\big(F^{\text{sew}}\big) \times \big(1+\Delta F^{\text{Z}}\big) \times \mathcal{M}_0$$

Electromagnetic terms:

$[\mu_{\text{R}}^2 = M_{\text{W}}^2]$

↪ correspond to (QED with $M_\gamma = 0$) minus (QED with $M_\gamma = M_{\text{W}}$)

$$\Delta F^{\text{em}} = \frac{1}{2}\sum_{i=1}^{n}\Bigg\{-\frac{\alpha}{4\pi}\sum_{j\neq i} Q_i Q_j \Big[I_{ij}(\epsilon,0) - I_{ij}(\epsilon,M_{\text{W}})\Big] + \left(\frac{\alpha}{4\pi}\right)^2 b^{(1)}_{\text{QED}} Q_i^2 \Big[J_{ii}(\epsilon,0,M_{\text{W}}^2) - J_{ii}(\epsilon,M_{\text{W}},M_{\text{W}}^2)\Big]\Bigg\},$$

$$I_{ij}(\epsilon,0) \overset{\text{NLL}}{=} -\left[3 - 2\ln\left(\frac{-r_{ij}}{Q^2}\right)\right]\epsilon^{-1} + \Bigg\{\overbrace{-\delta_{i,0}\,\epsilon^{-2} + \delta_{i,\text{t}}}^{\text{dependence on fermion mass } m_i}\Bigg[L\epsilon^{-1} + \frac{1}{2}L^2 + \frac{1}{6}L^3\epsilon + \frac{1}{24}L^4\epsilon^2 + \left(\frac{1}{2} - \ln\left(\frac{m_i^2}{M_{\text{W}}^2}\right)\right)\left(\epsilon^{-1} + L + \frac{1}{2}L^2\epsilon + \frac{1}{6}L^3\epsilon^2\right)\Bigg] + (i\to j)\Bigg\} + \mathcal{O}(\epsilon^3),$$

$$J_{ij}(\epsilon,0,\mu^2) = \frac{1}{\epsilon}\left[I_{ij}(2\epsilon,0) - \left(\frac{Q^2}{\mu^2}\right)^{\epsilon} I_{ij}(\epsilon,0)\right]$$

Comparison to existing results

- previous results for form factor and angular-dependent NLLs reproduced and extended Denner, Melles, Pozzorini '03; Pozzorini '04
- agreement with general resummation predictions based on evolution equations Melles '00, '01
- agreement with SCET results Chiu, Golf, Kelley, Manohar '07, '08

Application to 4-fermion scattering

- neutral current $f\bar{f} \to f'\bar{f}'$: NLL-agreement with massless-fermion N^{3}LL calculation, additional fermion-mass effects B.J., Kühn, Penin, Smirnov '05
- charged current $f_1\bar{f}_2 \to f_3\bar{f}_4$: new NLL result
- also applicable to processes with fermions and gluons, e.g. $g\,g \to f\,\bar{f}$: gluons $=$ legs with zero EW quantum numbers

IV Summary & outlook

Massless and massive fermionic processes $f_1 f_2 \to f_3 \cdots f_n$ (+ gluons) with $(p_i + p_j)^2 \gg M_\mathrm{W}^2$ and different masses $M_\mathrm{W}^2 \sim M_\mathrm{Z}^2 \sim m_\mathrm{t}^2 \sim M_\text{Higgs}^2$:

- 2-loop EW NLL corrections in $D = 4 - 2\epsilon$ dimensions
- loop integrals calculated with two independent methods
- Yukawa contributions only in wave-function renormalization
- universal correction factors, electromagnetic singularities separable
- process-independent: applicable for $e^+e^- \to f\bar{f}$, Drell–Yan, $gg \to f\bar{f}$, ...

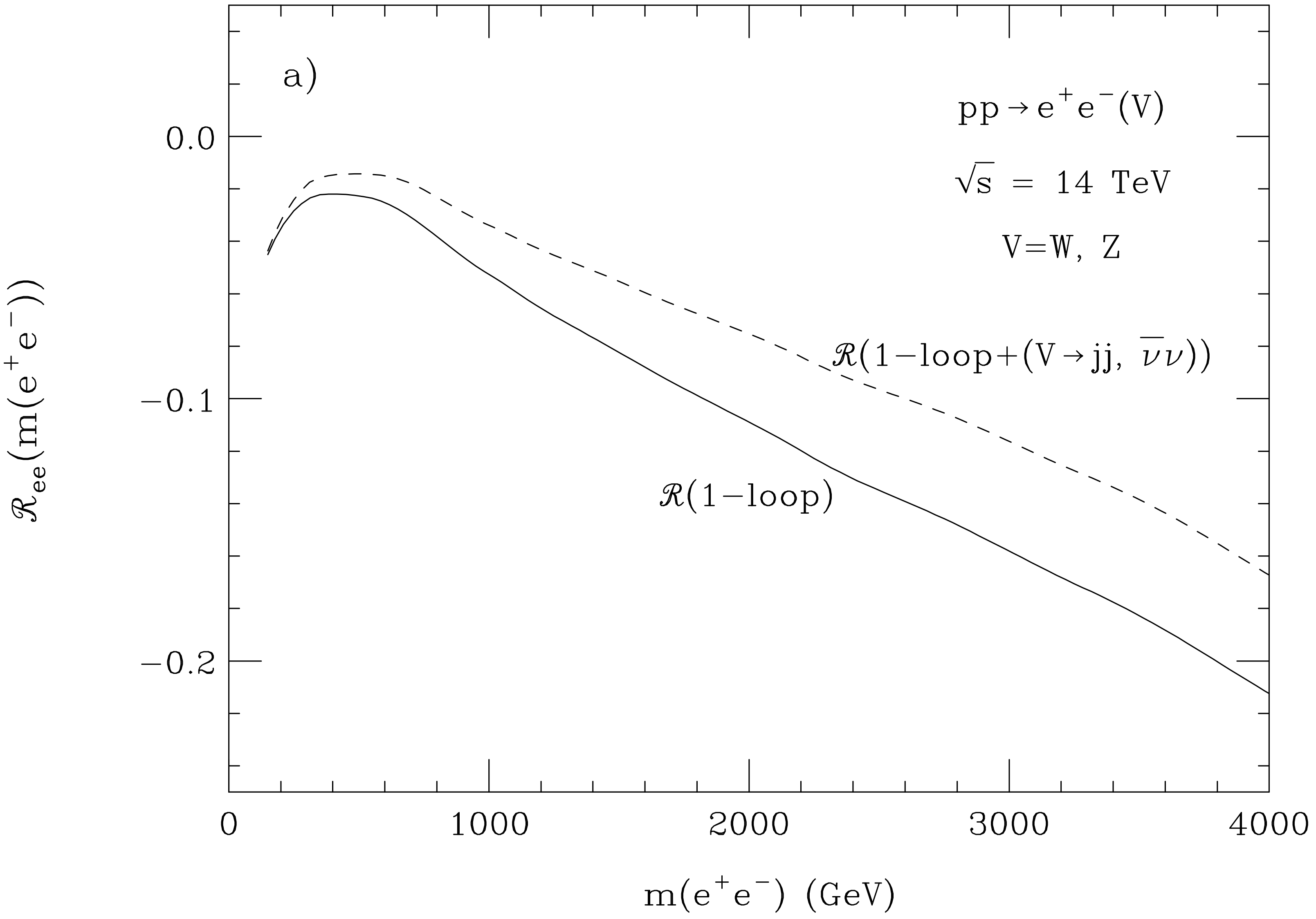
Outlook: EW corrections to arbitrary high-energy processes

- process-independent results for all Standard-Model particles possible at 1 loop

  Denner, Pozzorini '00, '01
- generalize 2-loop method for external gauge & Higgs bosons
- calculate relevant loop integrals ⇝ many already done for fermions

Extra slides

Virtual + Real W, Z emission: only partial cancellation



Baur, Phys. Rev. D75 (2007) 013005

Treatment of ultraviolet (UV) singularities

UV $1/\epsilon$ poles in subdiagrams with scale μ^2_{loop} & renormalization at scale μ^2_{R}:

$$\underbrace{\frac{1}{\epsilon}\left(\frac{Q^2}{\mu^2_{\text{loop}}}\right)^{\epsilon}}_{\text{bare diagrams}} - \underbrace{\frac{1}{\epsilon}\left(\frac{Q^2}{\mu^2_{\text{R}}}\right)^{\epsilon}}_{\text{counterterms}} = \ln\left(\frac{\mu^2_{\text{R}}}{\mu^2_{\text{loop}}}\right) + \mathcal{O}(\epsilon) \quad \Rightarrow \quad \text{possible NLL contribution}$$

Perform **minimal UV subtraction** in UV-singular (sub)diagrams and counterterms:

$$\underbrace{\frac{1}{\epsilon}\left[\left(\frac{Q^2}{\mu^2_{\text{loop}}}\right)^{\epsilon} - 1\right]}_{\text{bare diagrams}} - \underbrace{\frac{1}{\epsilon}\left[\left(\frac{Q^2}{\mu^2_{\text{R}}}\right)^{\epsilon} - 1\right]}_{\text{counterterms}}$$

Advantages:

- no UV NLL terms from hard subdiagrams ($\mu^2_{\text{loop}} \sim Q^2$)
  $\hookrightarrow$ no UV contributions from internal parts of tree subdiagrams
- can use soft–collinear approximation (not valid in UV regime!) also for hard UV-singular subdiagrams