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Non-resonant corrections to top-pair production in the threshold region at linear colliders

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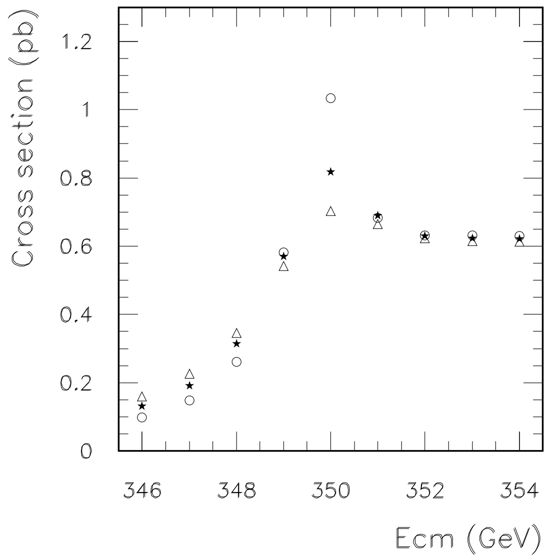
I Top-pair production at linear colliders near threshold

Future linear colliders (ILC/CLIC)
with $\sqrt{s} \gtrsim 2m_t \approx 350\,\text{GeV} \rightsquigarrow$ produce many $t\bar{t}$ pairs:
clean initial state of $e^+e^- \to t\bar{t}$ allows
threshold scans with $\sqrt{s} \sim 2m_t$

$\hookrightarrow$ precise determination of top-quark parameters $(m_t, \Gamma_t, \ldots)$,
especially as input for electroweak precision observables



Martinez, Miquel '02

Need also precise theoretical prediction!

QCD corrections are known (almost) up to NNNLO order,
but electroweak (EW) non-resonant contributions were missing even at NLO!

The **decay** $t\bar{t} \to (bW^+)\,(\bar{b}W^-)$ is an EW effect.
$\Rightarrow$ Describe $t\bar{t}$ production in terms of the more physical process $e^+e^- \to W^+W^-b\bar{b}$.
$\Rightarrow$ Allow for invariant-mass cuts on reconstructed $t, \bar{t}$.

Perturbative expansion: NRQCD

Decay $t \to bW^+$ with $\Gamma_t \approx 1.5\,\text{GeV} \gg \Lambda_{\text{QCD}} \Rightarrow t\bar{t}$ is perturbative at threshold.
Bigi, Dokshitzer, Khoze, Kühn, Zerwas '86

Top quarks move slowly near threshold: velocity $v \sim \alpha_s \ll 1$

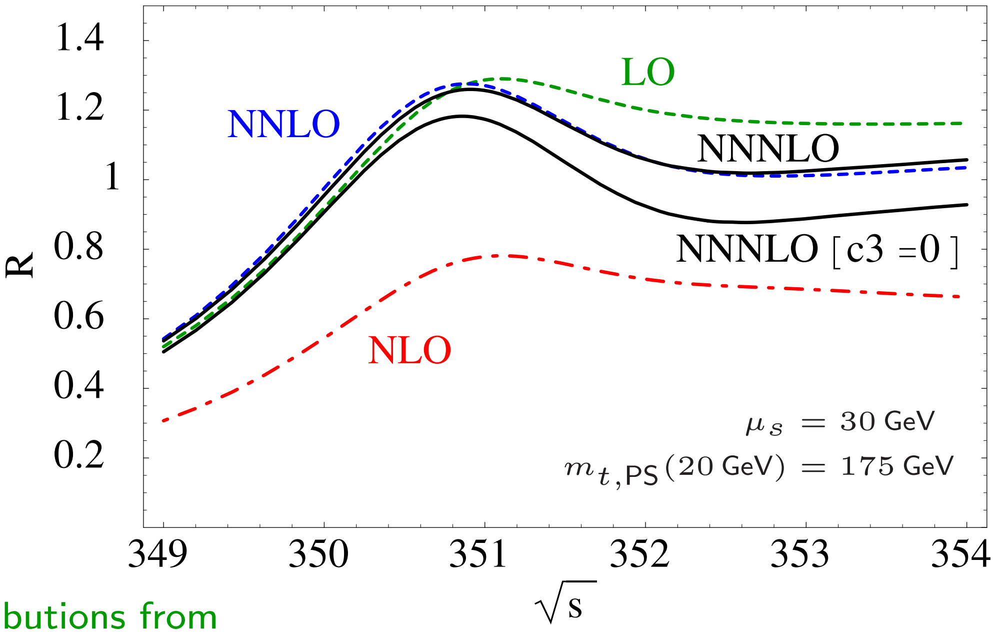
$\hookrightarrow$ sum $\left(\frac{\alpha_s}{v}\right)^n$ from "Coulomb gluons" to all orders

$\hookrightarrow$ expansion: LO, NLO, ... from additional powers of α_s or v

Further improvement by summing also $(\alpha_s \ln v)^m$ to all orders: LL, NLL, ...

Status of QCD corrections

- NNLO QCD corrections
  Hoang, Teubner '98–'99; Melnikov, Yelkhovsky '98; Yakovlev '98; Beneke, Signer, Smirnov '99; Nagano, Ota, Sumino '99; Penin, Pivovarov '98–'99
- NNLO & (partial) NNLL
  Hoang, Manohar, Stewart, Teubner '00–'01; Hoang '03; Pineda, Signer '06
- (partial) NNNLO
  Beneke, Kiyo, Schuller '05–'08 $\rightsquigarrow$ **see figure** [+ contributions from Kiyo, Seidel, Steinhauser '08; Anzai, Kiyo, Sumino '09; Smirnov, Smirnov, Steinhauser '09–'10]

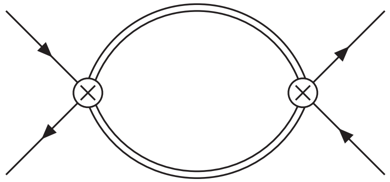


Effective field theory (EFT) for pair production of unstable particles near threshold

Beneke, Chapovsky, Khoze, Signer, Stirling, Zanderighi '01–'04;
Actis, Beneke, Falgari, Schwinn, Signer, Zanderighi '07–'08

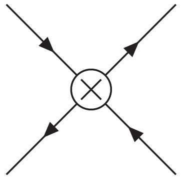
- Non-relativistic power counting: $\boxed{\alpha_s^2 \sim \alpha_{\mathrm{EW}} \sim \dfrac{\Gamma_t}{m_t} \sim v^2 = 1 - \dfrac{4m_t^2}{s}}$
- Integrate out hard modes $\sim m_t \rightsquigarrow$ EFT with potential (nearly on-shell) top quarks.
- Extract cross section $e^+e^- \to W^+W^-b\bar{b}$ from appropriate cuts of the $e^+e^- \to e^+e^-$ forward-scattering amplitude:

resonant contributions



with production operators of potential $t\bar{t}$ pair

non-resonant contributions



correspond to full-theory diagrams expanded around $\Gamma_t = 0$ and $s = 4m_t^2$

$\Rightarrow$ Potential corrections to resonant diagrams within EFT

$\Rightarrow$ Hard corrections to matching coefficients of operators

Electroweak effects at LO

- Replacement rule $E = \sqrt{s} - 2m_t \to E + i\Gamma_t$ — Fadin, Khoze '87
  ($\rightsquigarrow$ implemented in existing QCD corrections)

Electroweak effects at NLO

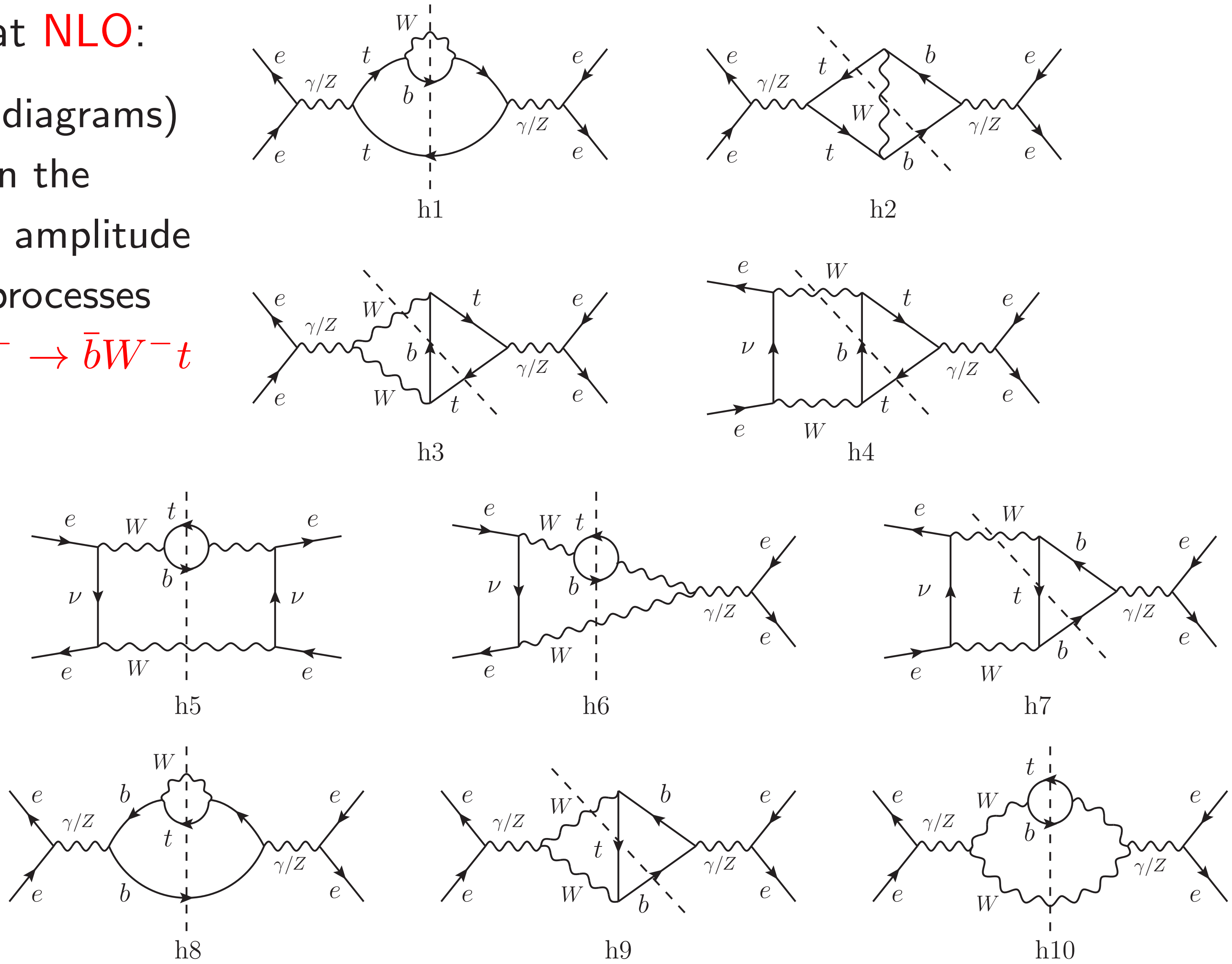
- Exchange of a "Coulomb photon": trivial extension of QCD corrections (available)
- Gluon exchange between top quarks and their decay products: — Fadin, Khoze, Martin '94; Melnikov, Yakovlev '94; Hoang, Reißer '05
  $\hookrightarrow$ cancel at NLO & NNLO in the total cross section.
  They are still negligible for *loose* top invariant-mass cuts.

- **Non-resonant (hard) corrections $\rightsquigarrow$ topic of this talk!**

II Electroweak non-resonant NLO contributions

Non-resonant corrections at NLO:

- cuts through $bW^+\bar{t}$ (see diagrams) and $\bar{b}W^-t$ (not shown) in the 2-loop forward-scattering amplitude
- correspond to tree-level processes $e^+e^- \to bW^+\bar{t}$ and $e^+e^- \to \bar{b}W^-t$
- hard region at NLO: $\Gamma_t = 0$ and $s = 4m_t^2$



[symmetric diagrams not shown]

Form of non-resonant contributions

With the reconstructed top momentum $p_t = p_b + p_{W^+}$ (top only present in h_1–h_4), the contributions of all diagrams (for $s = 4m_t^2$) are of the form:

$$\int_{\Delta^2}^{m_t^2} \mathrm{d}p_t^2 \, (m_t^2 - p_t^2)^{1/2-\epsilon} \, H_i\left(\frac{p_t^2}{m_t^2}, \frac{M_W^2}{m_t^2}\right)$$

Total cross section: $\Delta^2 = M_W^2$

Top invariant-mass cuts:

Restrict invariant masses $M_{t,\bar{t}}$ of the reconstructed $t, \bar{t}$: $\boxed{\left|M_{t,\bar{t}} - m_t\right| \leq \Delta M_t}$

$\hookrightarrow$ lower integration limit: $M_W^2 \leq \Delta^2 < m_t^2$

We focus on loose cuts with $\Delta M_t \gg \Gamma_t$

$\hookrightarrow$ no cut needed for resonant contributions.

III Results & comparisons

obtained with $m_t = 172\,\text{GeV}$ and $\Gamma_t = \Gamma_t^{\text{tree}} = 1.46550\,\text{GeV}$

Tree-level comparison to MadGraph/MadEvent/MadAnalysis (MG) Alwall et al. '07

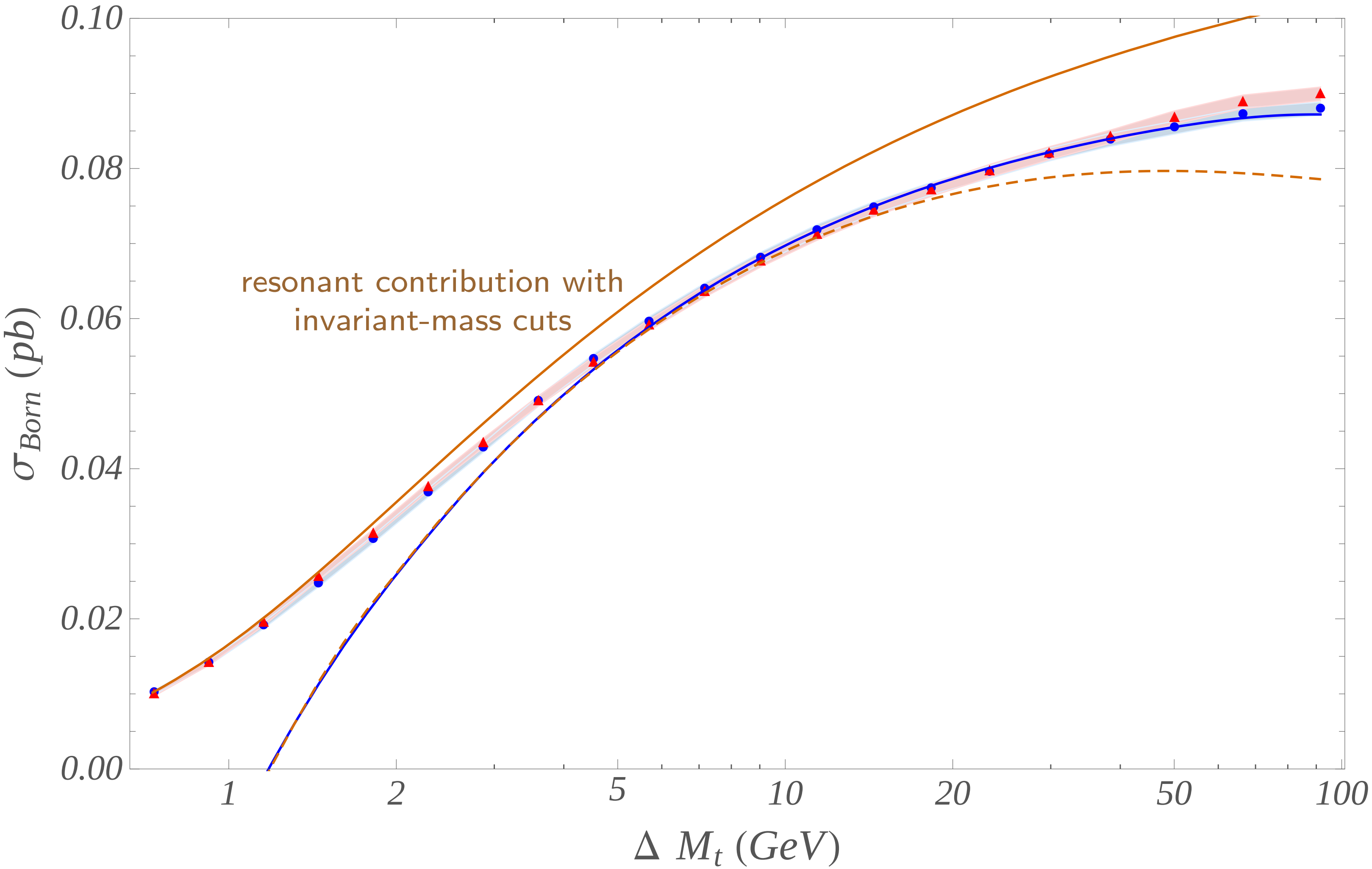
- generated 10^4 events for $e^+e^- \to W^+W^-b\bar{b}$,
- analyzed dependence on invariant-mass cuts

Comparison to recent alternative approach Hoang, Reißer, Ruiz-Femenía '10

- invariant-mass cuts through "phase-space matching" within non-relativistic EFT (QCD & EW @ NLO + some NNLO contributions)
- contributions are expanded for moderate invariant-mass cuts
  $15\,\text{GeV} \leq \Delta M_t \leq 35\,\text{GeV}$
  $\leftrightarrow$ our result is also valid for larger ΔM_t up to the total cross section.
- EW contributions match leading powers in $\Delta M_t/m_t$ of our result
  $\hookrightarrow$ agreement for small cut parameter ΔM_t

EW tree-level contributions: cut-dependence at threshold

cross section (for $\alpha_s = 0$) at threshold ($s = 4m_t^2$) as a function of the invariant-mass cut ΔM_t



Our result (solid-blue): EW non-resonant NLO + resonant NNLO tree-level contributions

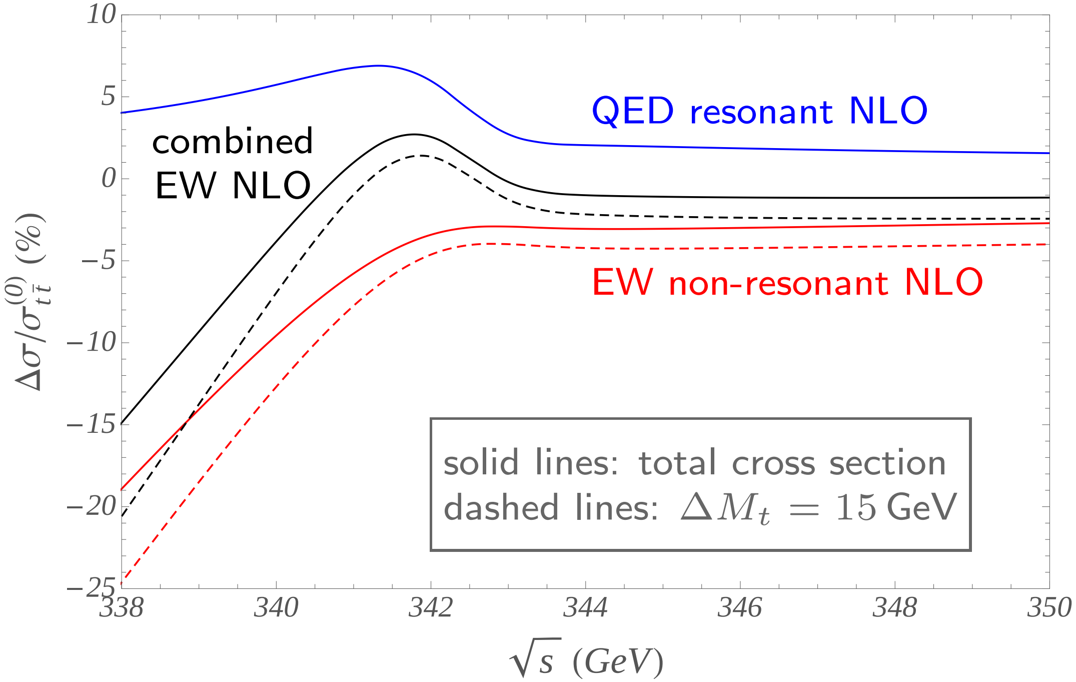
$\hookrightarrow$ good agreement with MadGraph (MG) for loose cuts $\Delta M_t \gtrsim 5\,\text{GeV}$

Hoang–Reißer–Ruiz-Femenía (HRR) result: dashed-brown $\Rightarrow$ agrees with our result for small ΔM_t

Resonant contribution with inv.-mass cuts (LO): solid-brown $\Rightarrow$ agrees with MG for tight cuts $\Delta M_t \lesssim \Gamma_t$

Full cross section with QCD LO & EW NLO contributions

$[\alpha_s^{\overline{\mathrm{MS}}}(30\,\mathrm{GeV}) = 0.142]$

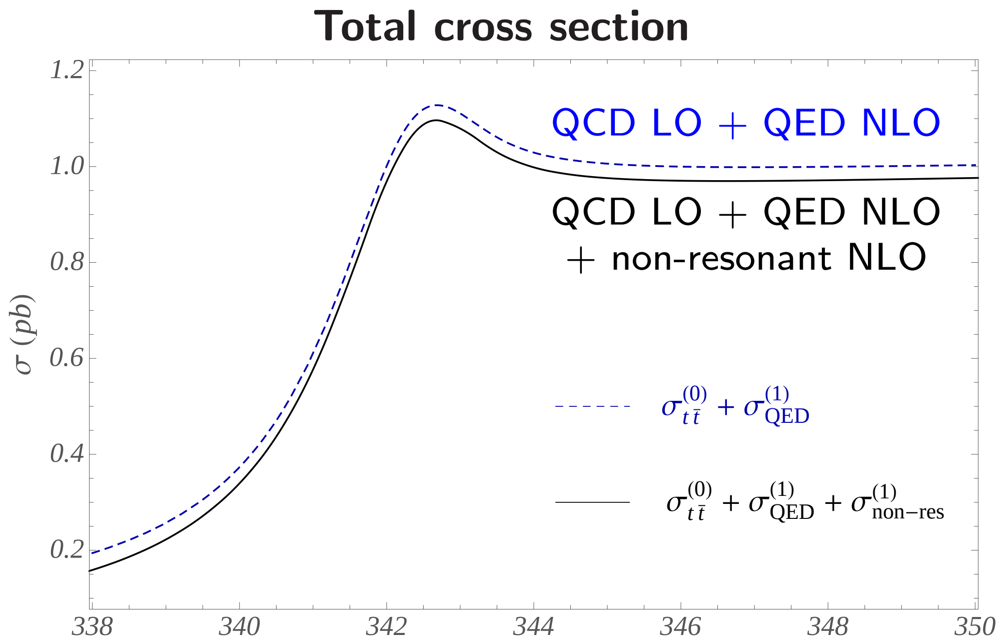


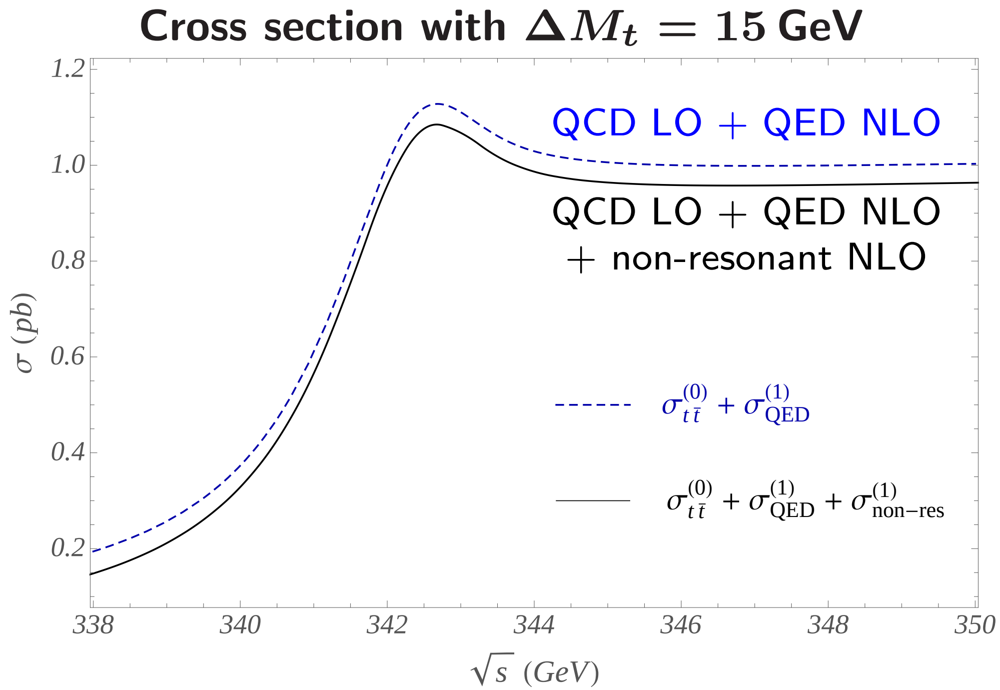
Relative sizes of EW NLO corrections w.r.t. LO (incl. resummed "Coulomb gluons"):

QED resonant correction ("Coulomb photons"),

non-resonant NLO correction,

combined EW NLO corrections





IV Singularities of NNLO contributions

Singularities from resonant contributions

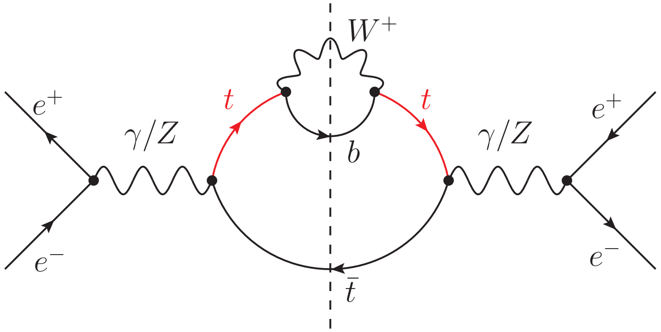
Divergences arise due to finite top width.

At **NNLO**: finite-width divergences $\propto \boxed{\alpha_s \frac{\Gamma_t}{\epsilon}}$ (in dimensional regularization)

$\hookrightarrow$ must be cancelled by non-resonant NNLO contributions.

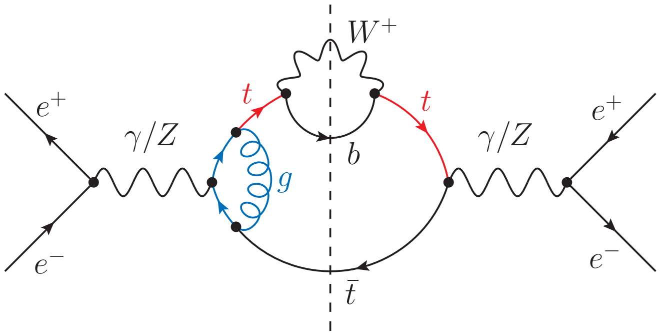
Singularities from non-resonant contributions

End-point divergences of the phase-space integration at $p_t^2 \to m_t^2$: (because $\Gamma_t = 0$)

NLO: 

$$\sim \int^{m_t^2} \frac{dp_t^2}{(m_t^2 - p_t^2)^{n+\epsilon}} \text{ with } n = \tfrac{3}{2}, \tfrac{1}{2}, \ldots$$

$\hookrightarrow$ end-point divergence <u>finite</u> in dim. reg. $\left[\int_{\Delta^2}^{m_t^2} \frac{dp_t^2}{(m_t^2 - p_t^2)^{\frac{3}{2}+\epsilon}} = -\frac{2}{(m_t^2 - \Delta^2)^{\frac{1}{2}}} + \mathcal{O}(\epsilon)\right]$

NNLO: 

$$\sim \int^{m_t^2} \frac{dp_t^2}{(m_t^2 - p_t^2)^{n+a\epsilon}} \text{ with } n = 2, \tfrac{3}{2}, \mathbf{1}, \tfrac{1}{2}, \ldots$$

$\hookrightarrow$ end-point divergence $\propto \boxed{\alpha_s \frac{\Gamma_t}{\epsilon}}$ from $n = 1$. $\left[\int_{\Delta^2}^{m_t^2} \frac{dp_t^2}{(m_t^2 - p_t^2)^{1+2\epsilon}} = -\frac{1}{2\epsilon} + \mathcal{O}(\epsilon^0)\right]$

$\Rightarrow$ **Extract NNLO end-point divergences** from gluon corrections to diagrams h_1–h_{10}.

Difficulties: • need loop & phase-space integration in $4 - 2\epsilon$ dimensions

• overlapping of end-point & soft–collinear divergences

$\Rightarrow$ We have already evaluated several contributions. Work in progress . . .

V Summary & outlook

Non-resonant contributions to $e^+e^- \to W^+W^-b\bar{b}$ in the $t\bar{t}$ resonance region

- NLO correction completed by EW non-resonant contributions for total cross section and with top invariant-mass cuts.
- $\Delta\sigma_{\text{tot}} \sim$ -30 fb (-3% above threshold, higher impact below), with invariant-mass cuts even larger correction.

$\hookrightarrow$ Can be added to existing QCD results to improve accuracy of theoretical prediction.

Singularities of NNLO contributions

- Finite-width divergences from resonant contributions must cancel with end-point divergences from non-resonant gluon corrections.
- Evaluating end-point divergences & checking cancellation: work in progress . . .

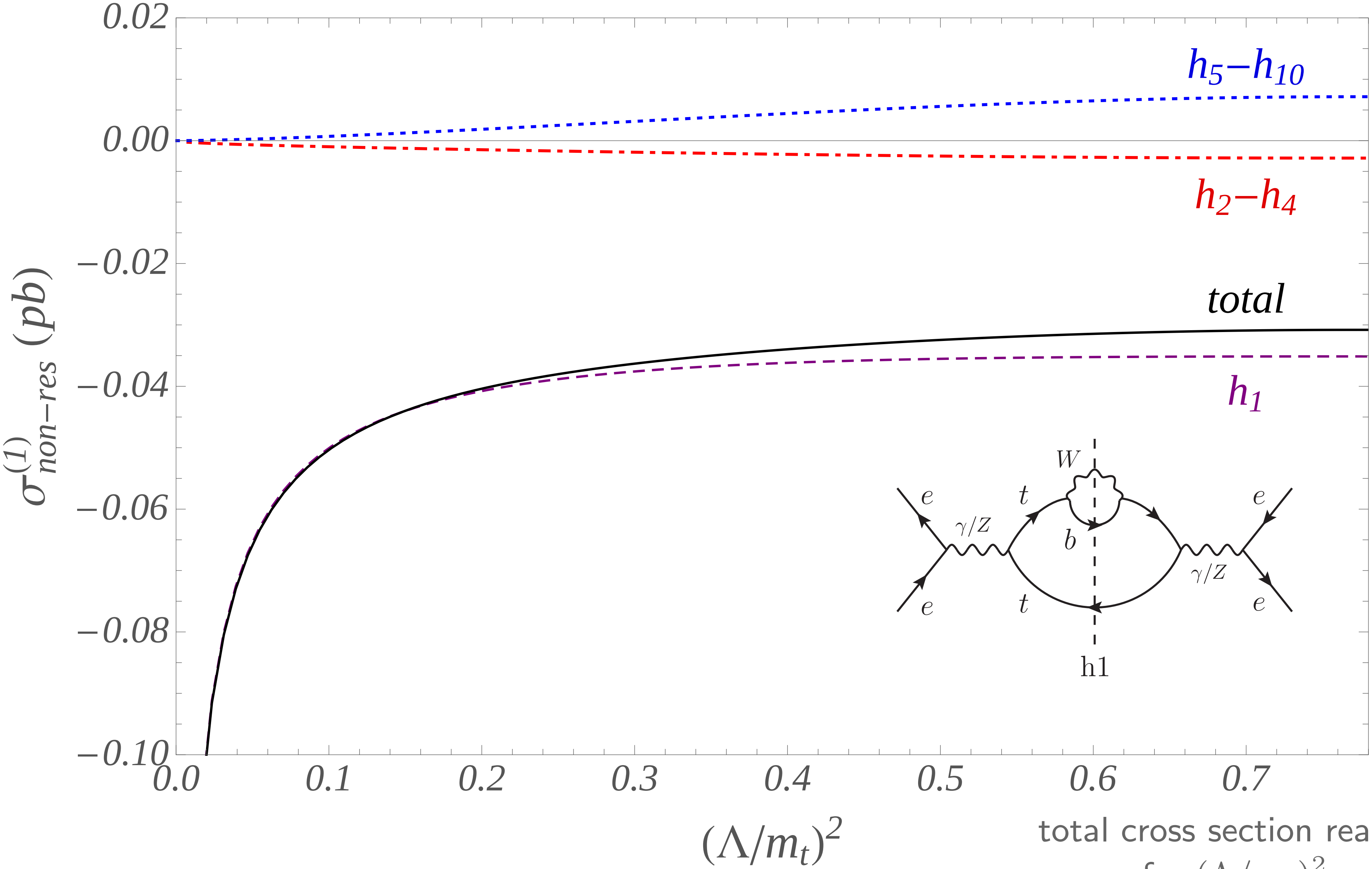
Outlook

Goal: calculate (complete/dominant?) NNLO non-resonant contributions.

Extra slides

Non-resonant corrections: contributions of the diagrams

contribution to cross section as a function of the invariant-mass cut $m_t^2 - p_t^2 \le \Lambda^2$

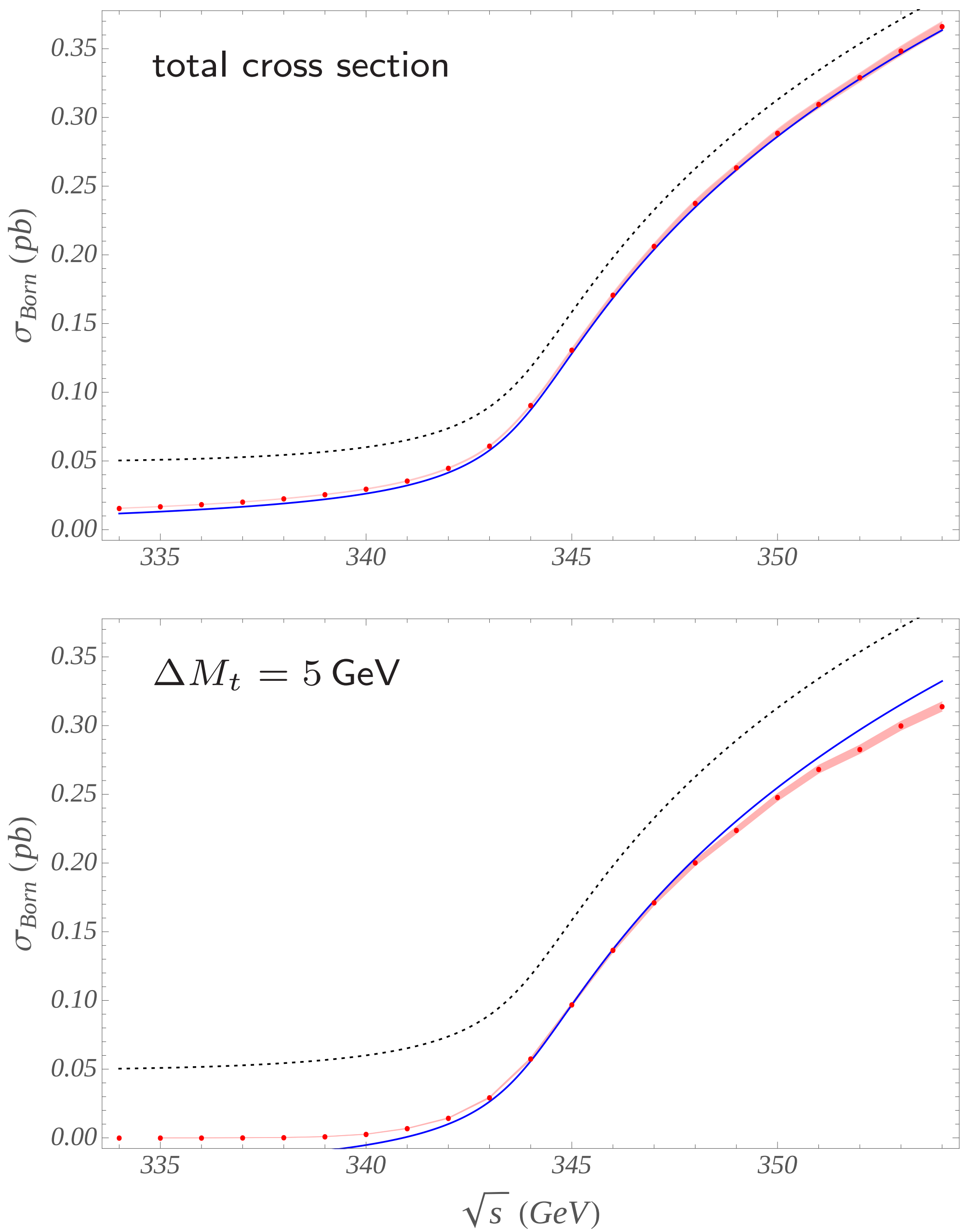


total cross section reached for $(\Lambda/m_t)^2 = 0.78$

$\hookrightarrow$ diagram h_1 yields dominant contribution

EW tree-level contributions: energy-dependence for different cuts

cross section (for $\alpha_s = 0$) as a function of the centre-of-mass energy $\sqrt{s}$



MG (full) points & error band,

EW NNLO tree-level contributions (solid-blue) [resonant + non-resonant],

only resonant contributions (dotted-black)