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Electroweak 2-loop corrections at high energies

The logarithmic form factor in a massive U(1) model and in a U(1)×U(1) model with mass gap

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Electroweak 2-loop corrections at high energies

I Why logarithmic 2-loop calculations in EW theory?

II Massive U(1) form factor

- evolution equation
- 2-loop results

III Methods for loop calculations at high energies

IV U(1)×U(1) model with mass gap

factorization of IR singularities

V Applications

how to treat the EW mass gaps Z – W – photon

VI Summary & outlook

towards 2-loop predictions for cross sections in EW theory

I Why logarithmic 2-loop calculations in EW theory?

Electroweak (EW) precision physics

- experimentally measured by now at energy scales up to $\sim M_{W,Z}$
- future generation of accelerators (LHC, ILC) $\rightarrow$ TeV region
- new energy domain $\sqrt{s} \gg M_{W,Z}$ becomes accessible

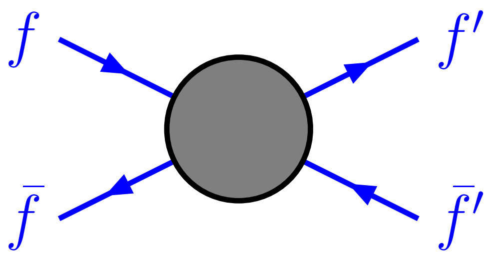
Electroweak radiative corrections at high energies $\sqrt{s} \sim$ TeV $\gg M_{W,Z}$

Fadin et al. '00; Kühn et al. '00, '01; Denner et al. '01, '03, '04; Pozzorini '04; B.F. et al. '03, '04; ...

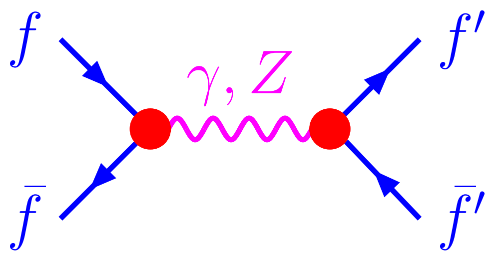
large negative corrections in *exclusive* cross sections

- EW corrections dominated by Sudakov logarithms $\alpha^n \ln^j(s/M^2_{W,Z})$, j = 2n, large coefficients in front of subleading logarithms $(0 \leq j < 2n)$
- 1-loop corrections $\gtrsim 10\%$
- 2-loop corrections $\gtrsim 1\%$, need to be under control for LHC/ILC
- single logarithmic contributions even larger, but strong cancellations

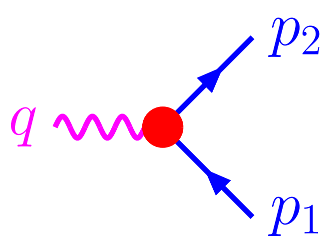
Important class of processes: 4-fermion scattering



$$A = \frac{ig^2}{s} F^2 \tilde{A}$$



Form factor F of vector current:



$$= F \cdot \bar{u}(p_2)\,\gamma^\mu\,u(p_1) + \underbrace{F' \cdot \bar{u}(p_2)\,\sigma^{\mu\nu}\,u(p_1)\,q_\nu}_{\text{vanishes when fermion masses are neglected}}$$

High energy behaviour $|s| \sim |t| \sim |u| \gg M_{W,Z}^2$

see Kühn et al. '01 for references

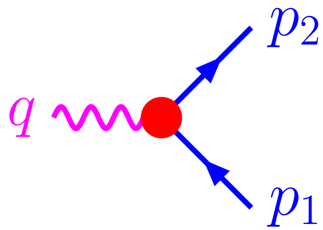
- all *collinear* logarithms of the amplitude A are part of the form factors F^2
- the *reduced amplitude* $\tilde{A}$ contains only *soft* logarithms
- $\tilde{A}$ satisfies an *evolution equation* known from massless calculations:

$$\frac{\partial \tilde{A}}{\partial \ln s} = \chi\big(\alpha(s)\big)\,\tilde{A}\,, \quad \chi = \text{matrix of soft anomalous dimensions}$$

$\Rightarrow$ still needed for 2-loop logarithms in A: form factor F

High energy behaviour of the form factor

$\hookrightarrow$ Sudakov limit:



$$= F(Q^2) \cdot \bar{u}(p_2)\, \gamma^\mu\, u(p_1)$$

- momentum transfer $-q^2 \equiv Q^2 \gg M^2 \equiv M^2_{W,Z}$

  $\left[\text{Euclidean } Q^2 > 0 \xrightarrow[\text{continuation}]{\text{analytic}} \text{Minkowskian } (-s) < 0\right]$

- neglect fermion masses $\longrightarrow$ external on-shell fermions: $p_1^2 = p_2^2 = 0$

- *logarithmic approximation*: neglect terms suppressed by a factor of M^2/Q^2

  $\hookrightarrow$ works well for 2-loop n_f contribution where the exact result in M^2/Q^2 is known

  B.F., Kühn, Moch '03

  $\Rightarrow$ contains constants and powers of the large logarithm $\ln(Q^2/M^2)$

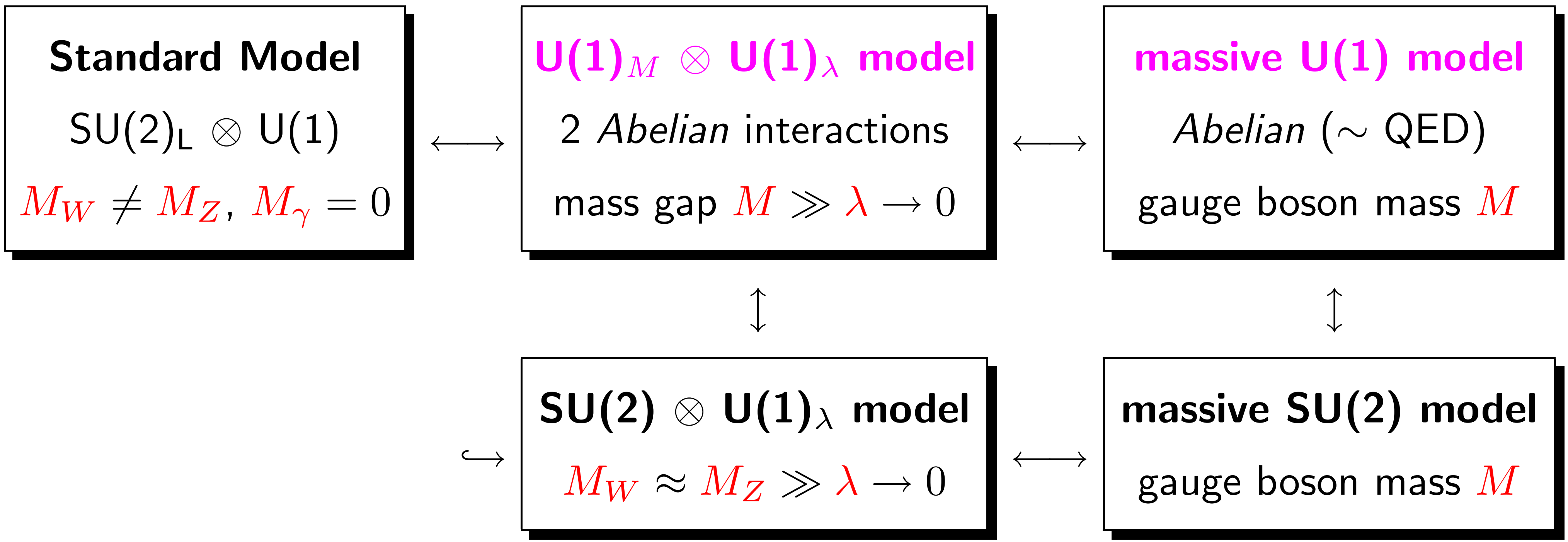
  $\Rightarrow$ leading order of asymptotic expansion in M^2/Q^2

Form factor and 4-fermion cross section have previously been known in NNLL accuracy at 2 loops: $\alpha^2 \ln^j(Q^2/M^2)$, $j = 4, 3, 2$

Kühn, Moch, Penin, Smirnov '01

Simplified models

1. decompose the problem into simpler parts:



2. use the partial results to compose a precise approximation of the Standard Model result

II Massive U(1) form factor

Form factor in perturbation theory: $F = 1 + \alpha\, F_1 + \alpha^2\, F_2 + \ldots$

large radiative corrections for $Q \sim$ TeV $\to$ sum up large logarithms to all orders in α:

$$F = 1 + \alpha\,(\ln^2 + \ln + \text{const}) + \alpha^2\,(\ln^4 + \ln^3 + \ln^2 + \ln + \text{const}) + \ldots$$
$$\leftrightarrow (1 + \alpha \cdot \text{const} + \alpha^2 \cdot \text{const} + \ldots)\,\exp\Big(\alpha\,(\ln^2 + \ln) + \alpha^2\,(\ln^3 + \ln^2 + \ln) + \ldots\Big)$$

Evolution equation in logarithmic approximation: Sen '81; Collins '89; Korchemsky '89; . . .

$$\frac{\partial F(Q^2)}{\partial \ln Q^2} = \left[\int_{M^2}^{Q^2} \frac{dx}{x}\gamma\big(\alpha(x)\big) + \zeta\big(\alpha(Q^2)\big) + \xi\big(\alpha(M^2)\big)\right] F(Q^2)$$

solution $\to$ exponentiation:

$$F(Q^2) = F_0\big(\alpha(M^2)\big)\exp\left\{\int_{M^2}^{Q^2} \frac{dx}{x}\left[\int_{M^2}^{x} \frac{dx'}{x'}\gamma\big(\alpha(x')\big) + \zeta\big(\alpha(x)\big) + \xi\big(\alpha(M^2)\big)\right]\right\}$$

Exponentiated form factor from the evolution equation:

$$F(Q^2) = F_0\big(\alpha(M^2)\big) \exp\left\{ \int_{M^2}^{Q^2} \frac{dx}{x} \left[\int_{M^2}^{x} \frac{dx'}{x'} \gamma\big(\alpha(x')\big) + \zeta\big(\alpha(x)\big) + \xi\big(\alpha(M^2)\big) \right] \right\}$$

perturbative expansion of the functions γ, ζ, ξ and F_0:

$$\gamma(\alpha) = \alpha\,\gamma_1 + \alpha^2\,\gamma_2 + \ldots \quad \text{etc.}$$

running of the coupling constant:

$$\alpha(x) = \alpha(M^2) - \ln\left(\frac{x}{M^2}\right) \frac{\beta_0}{4\pi}\, \alpha(M^2)^2 + \ldots$$

$\Rightarrow$ perform the integrals over x and x' in the exponent

$\hookrightarrow$ expansion of the exponent in α and powers of $\ln(Q^2/M^2)$

- compare the expansion of the exponentiated form factor to the perturbative result of a fixed order in α
- determine the corresponding coefficients of γ, ζ, ξ and F_0
- obtain a *leading logarithmic approximation* to all orders in α

Coefficients of γ, ζ, ξ and F_0 previously known for massive SU(N) and U(1) models:

- 1-loop result $\longrightarrow$ γ, ζ, ξ and F_0 up to $\mathcal{O}(\alpha)$
- massless 2-loop result $\longrightarrow$ γ up to $\mathcal{O}(\alpha^2)$ Kodaira, Trentadue '81

$$\gamma(\alpha) = -2C_F \frac{\alpha}{4\pi} \left\{ 1 + \frac{\alpha}{4\pi} \left[\left(\frac{67}{9} - \frac{\pi^2}{3} \right) C_A - \frac{20}{9} T_F n_f \right] \right\} + \mathcal{O}(\alpha^3)$$

$$\zeta(\alpha) = 3C_F \frac{\alpha}{4\pi} + \mathcal{O}(\alpha^2)$$

$$\xi(\alpha) = 0 + \mathcal{O}(\alpha^2)$$

$$F_0(\alpha) = -C_F \left(\frac{7}{2} + \frac{2}{3}\pi^2 \right) \frac{\alpha}{4\pi} + \mathcal{O}(\alpha^2)$$

- 1-loop running of α $\leftrightarrow$ 1-loop β-function:

$$\beta_0 = \frac{11}{3} C_A - \frac{4}{3} T_F n_f$$

$\Rightarrow$ NNLL approximation of 2-loop form factor F_2 known: $\alpha^2 \left(\ln^4 + \ln^3 + \ln^2 \right)$

Massive U(1) form factor in 2-loop approximation

known from evolution equation & full calculation of n_f contribution: ($n_f = \#$ fermions)

$$\alpha^2 F_2 = \left(\frac{\alpha}{4\pi}\right)^2 \left[+\frac{1}{2}\ln^4\left(\frac{Q^2}{M^2}\right) - \left(\frac{4}{9}n_f + 3\right)\ln^3\left(\frac{Q^2}{M^2}\right) + \left(\frac{38}{9}n_f + \frac{2}{3}\pi^2 + 8\right)\ln^2\left(\frac{Q^2}{M^2}\right) - \left(\frac{34}{3}n_f + \ldots\right)\ln\left(\frac{Q^2}{M^2}\right) + \left(\frac{16}{27}\pi^2 + \frac{115}{9}\right)n_f + \ldots \right]$$

Kühn, Moch, Penin, Smirnov '01
B.F., Kühn, Moch '03

- growing coefficients with alternating sign:

$$\begin{array}{ccccc} & -0.4\,n_f\ln^3 & +4.2\,n_f\ln^2 & -11.3\,n_f\ln & +18.6\,n_f \\ +0.5\ln^4 & -\;3\;\ln^3 & +\;14.6\;\ln^2 & -\;\ldots\;\ln & +\;\ldots \end{array}$$

- $Q \sim 1\,\text{TeV} \rightarrow +\ln^4 \sim -\ln^3 \sim +\ln^2$
  $\rightarrow$ strong cancellations between logarithmic terms

complete 2-loop corrections in logarithmic approximation necessary

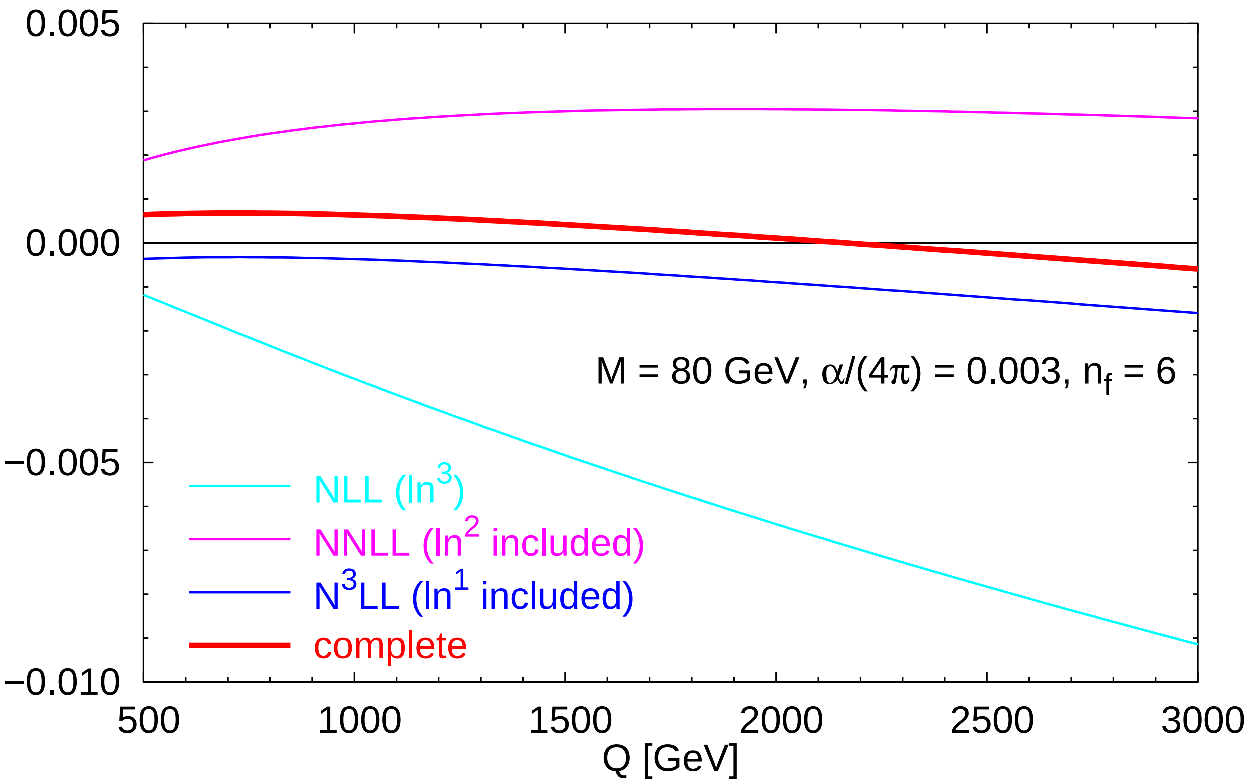
Massive U(1) form factor in 2-loop approximation: n_f part

successive logarithmic approximations:

B.F., Kühn, Moch, *Phys. Lett. B* 561 (2003) 111

$$\alpha^2 F_2 = n_f \left(\frac{\alpha}{4\pi}\right)^2 \left[-\frac{4}{9}\ln^3\left(\frac{Q^2}{M^2}\right) + \frac{38}{9}\ln^2\left(\frac{Q^2}{M^2}\right) - \frac{34}{3}\ln\left(\frac{Q^2}{M^2}\right) + \frac{16}{27}\pi^2 + \frac{115}{9}\right]$$

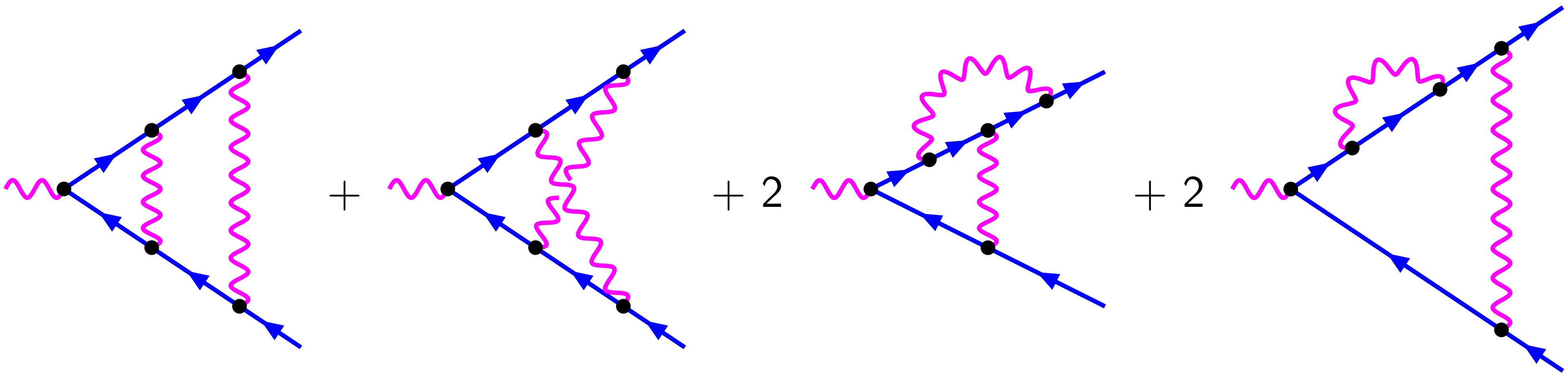
$+$ non-n_f part



- strong cancellations between logarithmic terms in n_f part
- good 2-loop approximation only with all logarithmic terms (and constant)
- behaviour of non-n_f part similar $\rightarrow$ need complete logarithmic approximation

Massive U(1) form factor in 2-loop approximation: diagrams $(n_f = 0)$

- complete 2-loop result $\longrightarrow$ loop calculation (*independent* of evolution equation)
- 2-loop vertex diagrams (massless fermions, massive bosons, 1 external scale):



+ external leg corrections + 1-loop × 1-loop

- reduction to scalar diagrams $\longrightarrow$ FORM (Vermaseren)
- scalar diagrams: expansion by regions
- evaluation of integrals and expansion in $\varepsilon = (4-d)/2 \longrightarrow$ Mathematica
- independent checks of all contributions

Massive U(1) form factor in 2-loop approximation: result $(n_f = 0)$

B.F., Kühn, Penin, Smirnov, *Phys. Rev. Lett.* 93 (2004) 101802

$$\alpha^2 F_2 = \left(\frac{\alpha}{4\pi}\right)^2 \Bigg[$$

$$+\frac{1}{2}\ln^4\left(\frac{Q^2}{M^2}\right)$$

agreement ✓

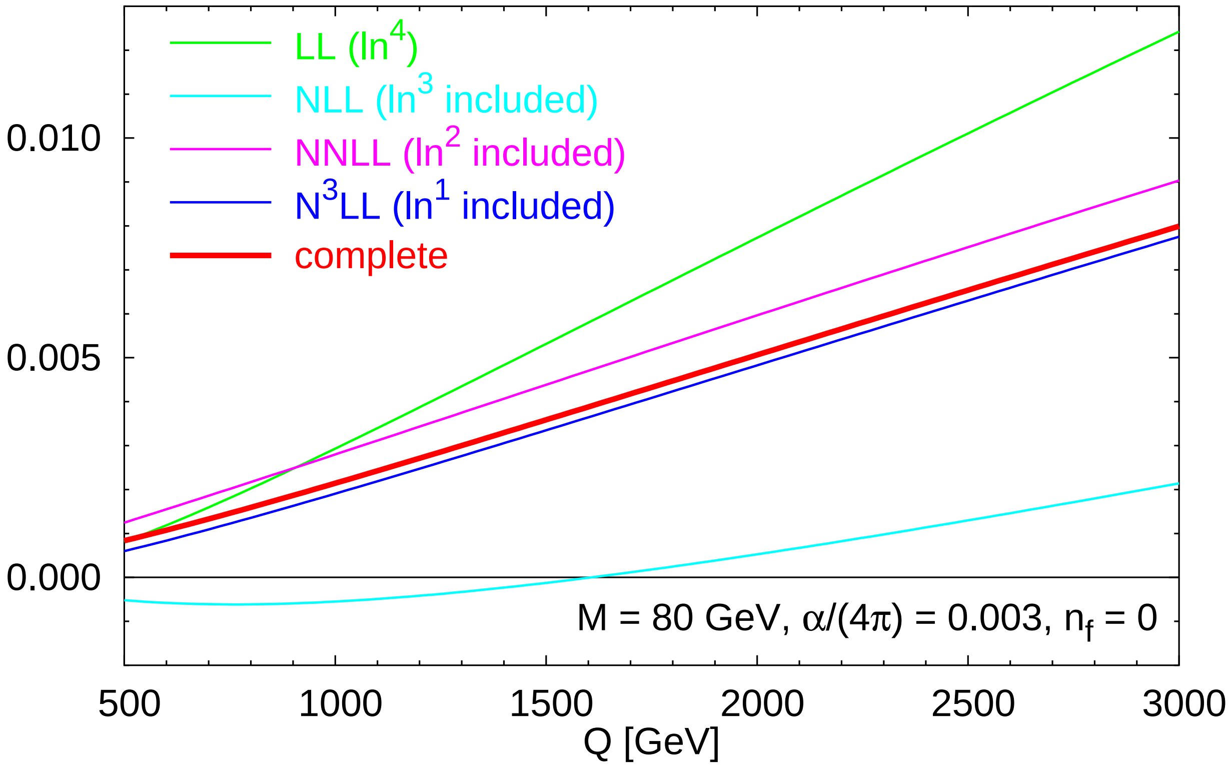
$$-3\ln^3\left(\frac{Q^2}{M^2}\right)$$

$$+\left(\frac{2}{3}\pi^2+8\right)\ln^2\left(\frac{Q^2}{M^2}\right)$$

$$-\left(-24\zeta_3+4\pi^2+9\right)\ln\left(\frac{Q^2}{M^2}\right)$$

$$+256\,\mathrm{Li}_4\left(\frac{1}{2}\right)+\frac{32}{3}\ln^4 2-\frac{32}{3}\pi^2\ln^2 2-\frac{52}{15}\pi^4+80\zeta_3+\frac{52}{3}\pi^2+\frac{25}{2}\Bigg]$$

new!



size of coefficients: $+0.5\ln^4 - 3\ln^3 + 14.6\ln^2 - 19.6\ln + 26.4$

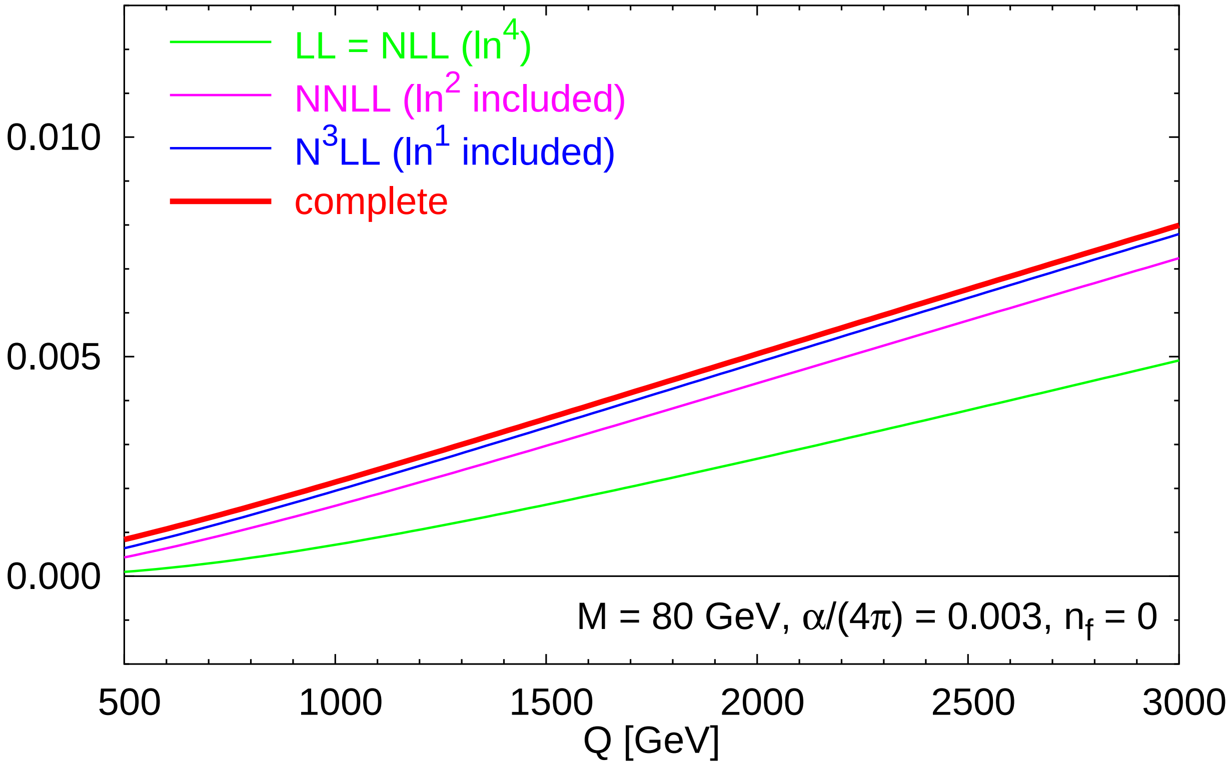
at $Q = 1$ TeV: $+326 \quad -387 \quad +372 \quad -99.2 \quad +26.4$

$\Rightarrow$ alternating signs! small constant (N^4LL) contribution

Remark: rescaling the argument of the logarithms, $M \to e^{3/4} M$

$$\ln\left(\frac{Q^2}{M^2}\right) \to \ln\left(\frac{Q^2}{(e^{3/4}M)^2}\right) + \frac{3}{2}$$

$\Rightarrow$ $\ln^3$ contribution vanishes!



size of coefficients after rescaling:	$+0.5\ln^4$	$+\,0\ln^3$	$+\,7.8\ln^2$	$+\,10.6\ln$	$+\,22.2$
at $Q = 1$ TeV:	$+79.5$	$+\,0$	$+\,98.8$	$+\,37.7$	$+\,22.2$

$\Rightarrow$ only positive signs!

Physical meaning?

III Methods for loop calculations at high energies

Reduction to scalar diagrams

- **given** from Feynman rules: $\mathcal{F}^\mu = \bar{u}(p_2)\,\Gamma^\mu(p_1,p_2)\,u(p_1)$
- **wanted:** form factor $F(Q^2)$ with $\mathcal{F}^\mu = F(Q^2)\cdot\bar{u}(p_2)\,\gamma^\mu\,u(p_1)$
- can be done using the properties of Dirac matrices and spinors,
  $\{\gamma^\mu,\gamma^\nu\} = 2g^{\mu\nu}$, $\not{p}_1\,u(p_1) = 0$, $\bar{u}(p_2)\,\not{p}_2 = 0$, combined with tensor reduction
- more elegantly with a *projector* on the form factor:

$$F(Q^2) = \frac{\mathrm{Tr}\left[\gamma_\mu\,\not{p}_2\,\Gamma^\mu(p_1,p_2)\,\not{p}_1\right]}{2(d-2)\,q^2}$$

- **output:** form factor $F(Q^2)$ in terms of *scalar Feynman integrals*

$$\int \mathrm{d}^d k_1 \int \mathrm{d}^d k_2\,\frac{\prod_{j=1}^{N}(\ell_j\cdot\ell'_j)^{\nu_j}}{\prod_{i=1}^{L}({k'_i}^2 - M_i^2)^{n_i}}$$

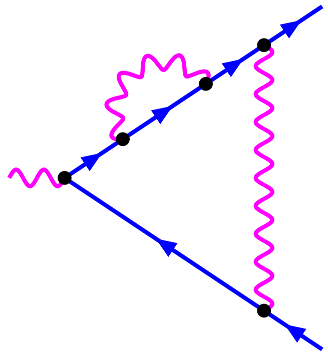
with L propagators and N irreducible scalar products in the numerator

Elimination of irreducible scalar products in the numerator

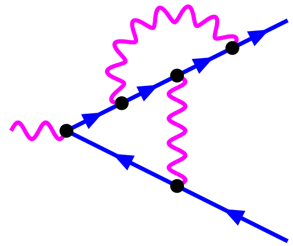
- Most scalar diagrams could directly be calculated *with numerator*.

- Diagrams with self-energy insertion:
  tensor reduction for inner loop, e.g.

$$\int \mathrm{d}^d k \, \frac{p \cdot k}{f(k,q)} = p_\nu \int \mathrm{d}^d k \, \frac{k^\nu}{f(k,q)} = \frac{p \cdot q}{q^2} \int \mathrm{d}^d k \, \frac{q \cdot k}{f(k,q)}$$



- Difficult diagrams where the absence of the numerator was desirable:



  ⋆ write propagators with Schwinger parameters (alpha parameters):

$$\frac{1}{(k^2 - M^2)^n} = \frac{1}{i^n \, \Gamma(n)} \int_0^\infty \mathrm{d}\alpha \, \alpha^{n-1} \, e^{i\alpha(k^2 - M^2)}$$

  ⋆ diagonalize the argument of the exponential in the loop momenta

  ⋆ perform tensor reduction: numerator → factors of $g^{\mu\nu}$

  ⋆ rewrite as linear combinations of the original integral *without numerator*,
  but with *higher powers of propagators* $(n \to n+1,\, n+2,\, \ldots)$
  and *higher dimension* $(d \to d+2,\, d+4,\, \ldots)$

Anastasiou et al. '00

Expansion by regions

a powerful method for the asymptotic expansion of Feynman diagrams — Beneke, Smirnov '98

- **given:** scalar Feynman integral & limit like $Q^2 \gg M^2$ *(Minkowskian limit!)*
- **wanted:** expansion of the *integral* in M^2/Q^2
- **problem:** direct expansion of the *integrand* leads to (new) IR/UV singularities

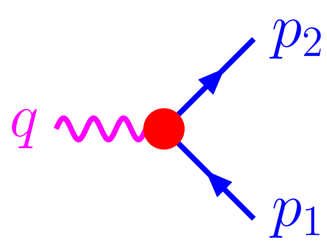
Recipe for the method of expansion by regions:

1. *divide* the integration domain into *regions* for the loop momenta (especially such regions where singularities are produced in the limit $M \to 0$)
2. in every region, *expand* the integrand in a *Taylor series* with respect to the parameters that are considered small *there*
3. *integrate* the expanded integrands over the *whole integration domain*
4. put to zero any *scaleless integral* (due to the properties of dimensional regularization)

- usually only a few regions give non-vanishing contributions
- for logarithmic approximation: only leading order of the expansion needed
  $\hookrightarrow$ in step 2. all small parameters in the integrand are simply set to zero
- sometimes additional regularization (apart from ε) needed for individual regions

Expansion by regions: example

Vertex form factor in the Sudakov limit $Q^2 \gg M^2$



- typical regions for each loop momentum k:

hard	(h):	all components of $k \sim Q$
soft	(s):	all components of $k \sim M$
ultrasoft	(us):	all components of $k \sim M^2/Q$
1-collinear	(1c):	$k^2 \sim 2p_1 \cdot k \sim M^2, \quad 2p_2 \cdot k \sim Q^2$
2-collinear	(2c):	$k^2 \sim 2p_2 \cdot k \sim M^2, \quad 2p_1 \cdot k \sim Q^2$

- 1-loop vertex correction: $f = \int \frac{\mathrm{d}^d k}{i\pi^{d/2}} \frac{1}{(k^2 - M^2)(k^2 - 2p_1 \cdot k)(k^2 - 2p_2 \cdot k)}$

$$f^{(h)} = \frac{1}{Q^2}\left[-\frac{1}{\varepsilon^2} + \frac{1}{\varepsilon}\ln(Q^2) - \frac{1}{2}\ln^2(Q^2) + \frac{\pi^2}{12} + \mathcal{O}\left(\frac{M^2}{Q^2}\right)\right]$$

$$f^{(1c)} + f^{(2c)} = \frac{1}{Q^2}\left[\frac{1}{\varepsilon^2} - \frac{1}{\varepsilon}\ln(Q^2) - \frac{1}{2}\ln^2(M^2) + \ln(M^2)\ln(Q^2) - \frac{5}{12}\pi^2 + \mathcal{O}\left(\frac{M^2}{Q^2}\right)\right]$$

$$\Rightarrow f = f^{(h)} + f^{(1c)} + f^{(2c)} = \frac{1}{Q^2}\left[-\frac{1}{2}\ln^2\left(\frac{Q^2}{M^2}\right) - \frac{\pi^2}{3} + \mathcal{O}\left(\frac{M^2}{Q^2}\right)\right]$$

Expansion by regions: why it works

simple $d = 1$ example: $f = \int_0^\infty \frac{\mathrm{d}k\, k^{-\varepsilon}}{(k+m)(k+q)}\,, \quad m \ll q$

$$\left.\begin{array}{ll} \text{soft (s):} & k < \Lambda \\ \text{hard (h):} & k > \Lambda \end{array}\right\} \text{where } m \ll \Lambda \ll q$$

$$\begin{aligned}
f &= \int_0^\Lambda \frac{\mathrm{d}k\, k^{-\varepsilon}}{(k+m)(k+q)} + \int_\Lambda^\infty \frac{\mathrm{d}k\, k^{-\varepsilon}}{(k+m)(k+q)} \\
&= \sum_{j=0}^\infty \frac{(-1)^j}{q^{j+1}} \int_0^\Lambda \frac{\mathrm{d}k\, k^{-\varepsilon+j}}{k+m} + \sum_{i=0}^\infty (-m)^i \int_\Lambda^\infty \frac{\mathrm{d}k\, k^{-\varepsilon-i-1}}{k+q} \\
&= \sum_{j=0}^\infty \frac{(-1)^j}{q^{j+1}} \left(\int_0^\infty \frac{\mathrm{d}k\, k^{-\varepsilon+j}}{k+m} - \int_\Lambda^\infty \frac{\mathrm{d}k\, k^{-\varepsilon+j}}{k+m} \right) + \sum_{i=0}^\infty (-m)^i \left(\int_0^\infty \frac{\mathrm{d}k\, k^{-\varepsilon-i-1}}{k+q} - \int_0^\Lambda \frac{\mathrm{d}k\, k^{-\varepsilon-i-1}}{k+q} \right) \\
&= \underbrace{\sum_{j=0}^\infty \frac{(-1)^j}{q^{j+1}} \int_0^\infty \frac{\mathrm{d}k\, k^{-\varepsilon+j}}{k+m}}_{f^{(s)}} + \underbrace{\sum_{i=0}^\infty (-m)^i \int_0^\infty \frac{\mathrm{d}k\, k^{-\varepsilon-i-1}}{k+q}}_{f^{(h)}} - \sum_{i=0}^\infty (-m)^i \sum_{j=0}^\infty \frac{(-1)^j}{q^{j+1}} \underbrace{\int_0^\infty \mathrm{d}k\, k^{-\varepsilon-i+j-1}}_{\to 0,\ \text{scaleless integral}} \\
&= f^{(s)} + f^{(h)} \quad \checkmark \\
&= \frac{\Gamma(\varepsilon)\Gamma(1-\varepsilon)}{(q-m)\, m^\varepsilon} - \frac{\Gamma(\varepsilon)\Gamma(1-\varepsilon)}{(q-m)\, q^\varepsilon} = \frac{\ln(q/m)}{q-m} + \mathcal{O}(\varepsilon) \quad \checkmark
\end{aligned}$$

Parameterization of Feynman integrals

- Feynman parameters:

$$\prod_i \frac{1}{A_i^{n_i}} = \frac{\Gamma(\sum_i n_i)}{\prod_i \Gamma(n_i)} \left(\prod_i \int_0^1 \mathrm{d}x_i \, x_i^{n_i-1} \right) \frac{\delta(\sum_i x_i - 1)}{(\sum_i x_i A_i)^{\sum_i n_i}}$$

- Schwinger parameters $\longrightarrow$ more general esp. with expansion by regions:

$$\frac{1}{A^n} = \frac{1}{i^n \, \Gamma(n)} \int_0^\infty \mathrm{d}\alpha \, \alpha^{n-1} \, e^{i\alpha A} \,, \quad \text{numerator } A^n = \left(\frac{1}{i} \frac{\partial}{\partial \alpha} \right)^n e^{i\alpha A} \Big|_{\alpha=0}$$

$\Rightarrow$ any number of propagators and numerators may be combined

$\Rightarrow$ can always be transformed to Feynman parameters

$\hookrightarrow$ evaluation:

$$\int \mathrm{d}^d k \, e^{i(\alpha k^2 + 2p\cdot k)} = i\pi^{d/2} \, (i\alpha)^{-d/2} \, e^{-ip^2/\alpha}$$

$$\int_0^\infty \mathrm{d}\alpha \, \alpha^{n-1} \, e^{i\alpha A} = \frac{i^n \, \Gamma(n)}{A^n}$$

$$\int_0^\infty \frac{\mathrm{d}\alpha \, \alpha^{n-1}}{(A + \alpha B)^r} = \frac{\Gamma(n) \, \Gamma(r-n)}{\Gamma(r) \, A^{r-n} \, B^n}$$

Mellin-Barnes representation

Feynman integrals with many scales / many massive propagators are hard to evaluate
$\hookrightarrow$ separate scales by Mellin-Barnes representation:

$$\frac{1}{(A+B)^n} = \frac{1}{\Gamma(n)} \int_{-i\infty}^{i\infty} \frac{\mathrm{d}z}{2\pi i}\, \Gamma(-z)\, \Gamma(n+z)\, \frac{B^z}{A^{n+z}}$$

- Mellin-Barnes integrals go along the imaginary axis, leaving poles of $\Gamma(-z+\ldots)$ to the right and poles of $\Gamma(z+\ldots)$ to the left of the integration contour
- applicable to massive propagators ($A = k^2$, $B = -M^2$) or to any complicated intermediate expression
- evaluation:
  close the integration contour to the right ($|B| \leq |A|$) or to the left ($|B| \geq |A|$) and pick up the residues within the contour using $\mathrm{Res}\, \Gamma(z)\big|_{z=-i} = (-1)^i / i!$
  $\Rightarrow$ *sums over Γ-functions*
  $\Rightarrow$ *multiple ζ-values* / *generalized (harmonic) polylogarithms* etc.
- close link to *expansion by regions*:
  Mellin-Barnes representation of the full integral
  $\hookrightarrow$ contributions corresponding to the regions

IV U(1)×U(1) model with mass gap

EW theory: massive and massless gauge bosons

$\hookrightarrow$ consider U(1)$_M$×U(1)$_\lambda$ model with 2 different masses $M \gg \lambda \to 0$

- pure U(1)$_M$: form factor $F(\alpha, Q, M)$
- pure U(1)$_\lambda$: form factor $F(\alpha', Q, \lambda)$
  $\to$ known from massive U(1) result ($M \to \lambda$, $\alpha \to \alpha'$)
  $\to$ IR (soft/collinear) singularities regularized by λ (or by poles in ε if $\lambda = 0$)
- combined U(1)$_M$×U(1)$_\lambda$: $\hat{F}(\alpha, \alpha', Q, M, \lambda)$
  $Q \gg M \gg \lambda \to$ **factorization of IR singularities:**

$$\hat{F}(\alpha, \alpha', Q, M, \lambda) = \underbrace{F(\alpha', Q, \lambda)}_{\text{IR singular}} \underbrace{\tilde{F}(\alpha, \alpha', Q, M)}_{\text{IR finite}} + \mathcal{O}\left(\alpha\alpha' \frac{\lambda^2}{M^2}\right)$$

Factorization of U(1)×U(1) form factor: results $(n_f = 0)$

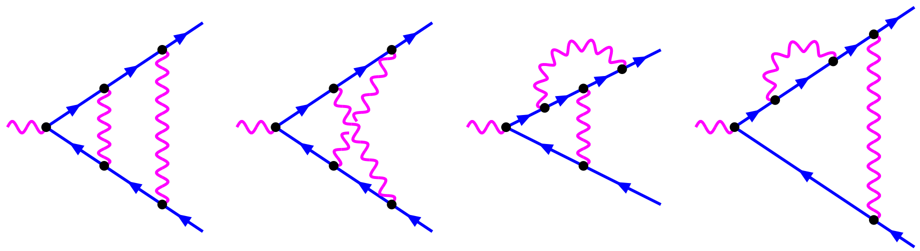
$$\hat{F}(\alpha,\alpha',Q,M,\lambda) = F(\alpha',Q,\lambda)\,\tilde{F}(\alpha,\alpha',Q,M) + \mathcal{O}\left(\alpha\alpha'\frac{\lambda^2}{M^2}\right)$$

$$\Rightarrow \tilde{F}(\alpha,\alpha',Q,M) = \lim_{\lambda\to 0}\frac{\hat{F}(\alpha,\alpha',Q,M,\lambda)}{F(\alpha',Q,\lambda)} = \lim_{\varepsilon\to 0}\frac{\hat{F}_\varepsilon(\alpha,\alpha',Q,M,0)}{F_\varepsilon(\alpha',Q,0)}$$

$\hookrightarrow$ set $\lambda = 0$ and calculate $\hat{F}_\varepsilon(\alpha,\alpha',Q,M,0)$ in dimensional regularization

calculation of 2-loop diagrams with

1 massive and 1 massless gauge boson:



$$\tilde{F}(\alpha,\alpha',Q,M) = F(\alpha,Q,M)\times$$

$$\left\{1 + \frac{\alpha\alpha'}{(4\pi)^2}\left[\underbrace{\left(48\zeta_3 - 4\pi^2 + 3\right)}_{21.2}\ln\left(\frac{Q^2}{M^2}\right) + \underbrace{\frac{7}{45}\pi^4 - 84\zeta_3 + \frac{20}{3}\pi^2 - 2}_{-22.0}\right]\right\}$$

$\Rightarrow$ interference terms are finite $\rightsquigarrow$ IR singularities factorize

$\Rightarrow$ additional terms contain only single logarithm $\ln^1$

Factorization of U(1)×U(1) form factor for $\boldsymbol{\lambda} = M$

$$\hat{F}(\alpha,\alpha',Q,M,\lambda) = F(\alpha',Q,\lambda)\,\tilde{F}(\alpha,\alpha',Q,M) + \mathcal{O}\left(\alpha\alpha'\frac{\lambda^2}{M^2}\right)$$

form of the suppressed interference terms $\mathcal{O}\left(\alpha\alpha'\frac{\lambda^2}{M^2}\right)$?

$\hookrightarrow$ set $\lambda = M$ and parameterize:

$$\hat{F}(\alpha,\alpha',Q,M,M) = F(\alpha',Q,M)\,\tilde{F}(\alpha,\alpha',Q,M)\,C(\alpha,\alpha',Q,M)$$

on the other hand: $\hat{F}(\alpha,\alpha',Q,M,M) = F(\alpha+\alpha',Q,M)$

$\hookrightarrow$ known from massive U(1) result $\longrightarrow$ calculate matching coefficient:

$$C(\alpha,\alpha',Q,M) = 1 + \frac{\alpha\alpha'}{(4\pi)^2}\underbrace{\left[512\,\mathrm{Li}_4\left(\tfrac{1}{2}\right) + \frac{64}{3}\ln^4 2 - \frac{64}{3}\pi^2\ln^2 2 - \frac{113}{15}\pi^4 + 244\zeta_3 + \frac{70}{3}\pi^2 + \frac{59}{4}\right]}_{-26.8}$$

$\Rightarrow$ interference term is constant, no logarithm

$\Rightarrow$ product $F(\alpha',Q,\lambda)\,\tilde{F}(\alpha,\alpha',Q,M)$ approaches $\hat{F}(\alpha,\alpha',Q,M,M)$ continuously for $\lambda \longrightarrow M$ with $\mathrm{N^3LL}$ accuracy!

V Applications

U(1)×U(1) form factor with mass gap from 1-mass result

massive W, Z & massless photon → need form factor with mass gap

suppose we cannot calculate $\hat{F}(\alpha, \alpha', Q, M, \lambda \to 0)$,
but we know $F(\alpha, Q, M)$ and $F(\alpha', Q, \lambda \to 0)$

↪ use $F(\alpha + \alpha', Q, M) = F(\alpha', Q, M)\, \tilde{F}(\alpha, \alpha', Q, M) + \mathcal{O}\big(\alpha\alpha' \ln^0\big)$

so we can get all logarithms in 2 loops:

$$\hat{F}(\alpha, \alpha', Q, M, \lambda \to 0) = F(\alpha', Q, \lambda \to 0)\, \frac{F(\alpha + \alpha', Q, M)}{F(\alpha', Q, M)} + \mathcal{O}\big(\alpha\alpha' \ln^0\big)$$

⇒ the calculation is reduced to the 1-mass case (with photon as heavy as W, Z)

Note:

SU(2)×U(1) model with mass gap → result only up to $\mathcal{O}\big(\alpha\alpha' \ln^1\big)$

Expanding the U(1)×U(1) form factor in a small mass difference

up to now, all heavy gauge bosons → same mass M,
but we need also $M_W \approx M_Z \to \lambda \approx M$:

$$\hat{F}(\alpha,\alpha',Q,M,\lambda) = F(\alpha',Q,\lambda)\,\tilde{F}(\alpha,\alpha',Q,M) + \underbrace{\mathcal{O}\Bigl(\alpha\alpha'\frac{\lambda^2}{M^2}\Bigr)}_{\mathcal{O}(\alpha\alpha'\ln^{0,1}),\ \lambda\to M}$$

↪ expand first term in $\delta \equiv \dfrac{M-\lambda}{M}$ for $\lambda \approx M$:

$$\hat{F}(\alpha,\alpha',Q,M,\lambda)\Big|_{\lambda\approx M} = F(\alpha+\alpha',Q,M)\cdot\Bigl\{1-\delta\frac{\alpha'}{4\pi}\Bigl[4\ln\Bigl(\frac{Q^2}{M^2}\Bigr)-6\Bigr]+\mathcal{O}(\delta^2)\Bigr\} + \mathcal{O}\bigl(\delta\,\alpha\alpha'\ln^{0,1}\bigr)$$

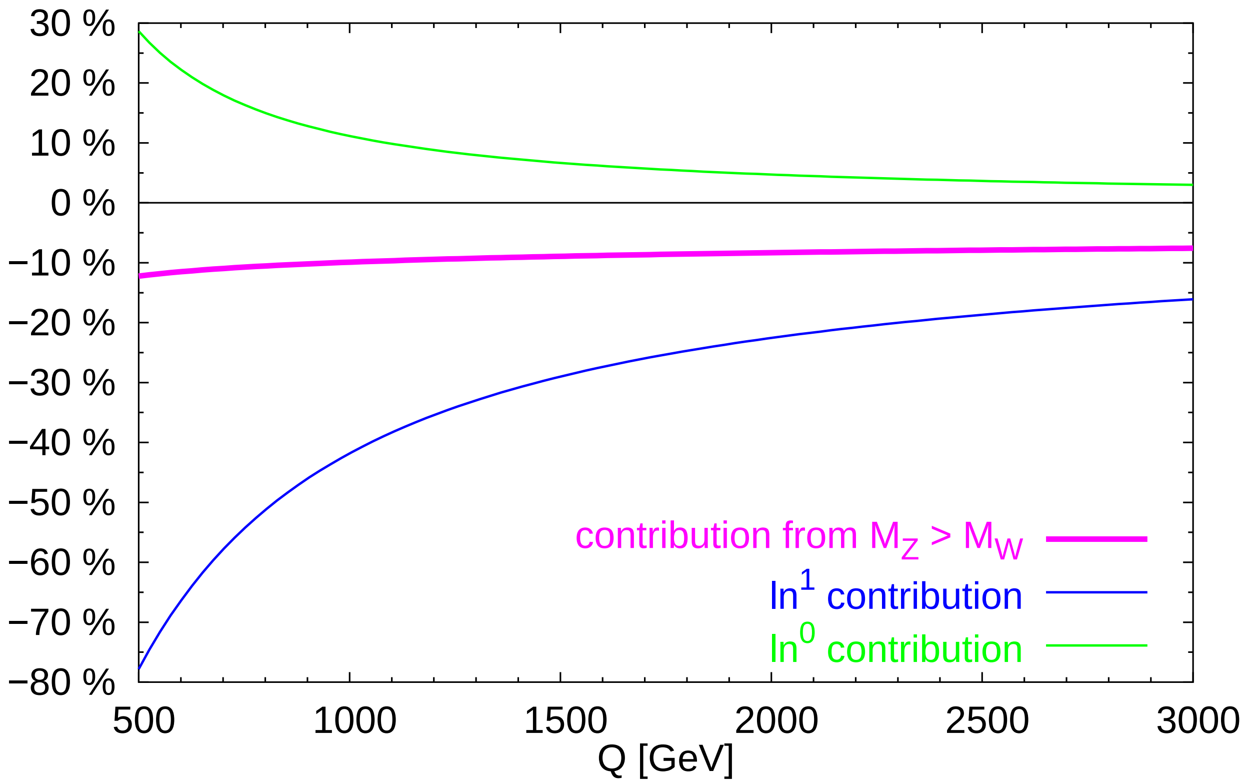
contribution of the mass difference to the form factor at order α^2 (for $\alpha' = \alpha$):

$$\Delta F\Big|_{\delta,\alpha^2} = -\delta\left(\frac{2\alpha}{4\pi}\right)^2\left[-2\ln^3\Bigl(\frac{Q^2}{M^2}\Bigr)+9\ln^2\Bigl(\frac{Q^2}{M^2}\Bigr)\underbrace{-\Bigl(16+\frac{4}{3}\pi^2\Bigr)}_{-29.2}\ln\Bigl(\frac{Q^2}{M^2}\Bigr)+\ldots\right]$$

Contribution of the $M_Z \neq M_W$ mass difference to the 2-loop form factor

$$M_W = 80.4\,\text{GeV}$$
$$M_Z = 91.2\,\text{GeV}$$

Relative contribution (in %)
of the mass difference $M_Z \neq M_W$
to the 2-loop form factor F_2



For comparison:

in blue/green: relative contribution of the linear logarithm / constant terms in F_2

$\Rightarrow$ The $M_Z \neq M_W$ mass difference can be taken into account by an expansion around the equal mass approximation.

VI Summary & outlook

Massive U(1) form factor

- simple model with massive gauge bosons
- complete 2-loop result in logarithmic approximation ✓

⇒ precise control of radiative corrections

U(1)×U(1) model with mass gap

- step towards EW theory with massive & massless gauge bosons
- factorization of IR singularities shown explicitly ✓

Applications

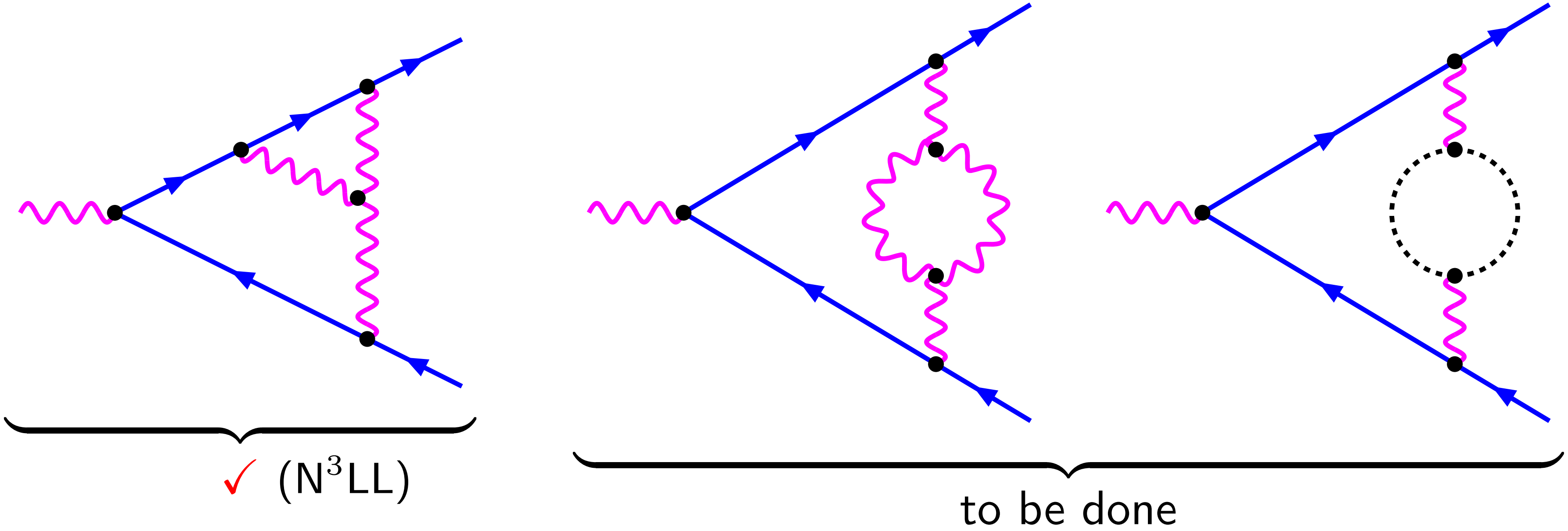
- calculation with mass gap reduced to the 1-mass case $M_W = M_Z = M_{\text{photon}}$
- $M_Z \neq M_W$ taken into account by expanding around the equal mass approximation

Various methods for loop calculations at high energies, e.g.

- expansion by regions → asymptotic expansion for Sudakov limits
- Mellin-Barnes representation, . . .

Outlook

- extend to non-Abelian models: SU(2), SU(N), SU(2)×U(1): *work in progress*



- consider Higgs contributions
- 4-fermion scattering amplitude
- predictions for EW corrections to $f\bar{f} \to f'\bar{f}'$ cross sections