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Structure of two-loop electroweak logarithmic corrections

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I Electroweak corrections at high energies

Electroweak (EW) collider physics

- up to now (LEP, Tevatron) at energy scales $\lesssim M_{\mathrm{W,Z}}$
- future colliders (LHC, ILC/CLIC) $\rightarrow$ reach TeV regime
  $\hookrightarrow$ new energy domain $\sqrt{s} \gg M_{\mathrm{W}}$ becomes accessible!

EW radiative corrections at high energies $\sqrt{s} \gg M_{\mathrm{W}}$

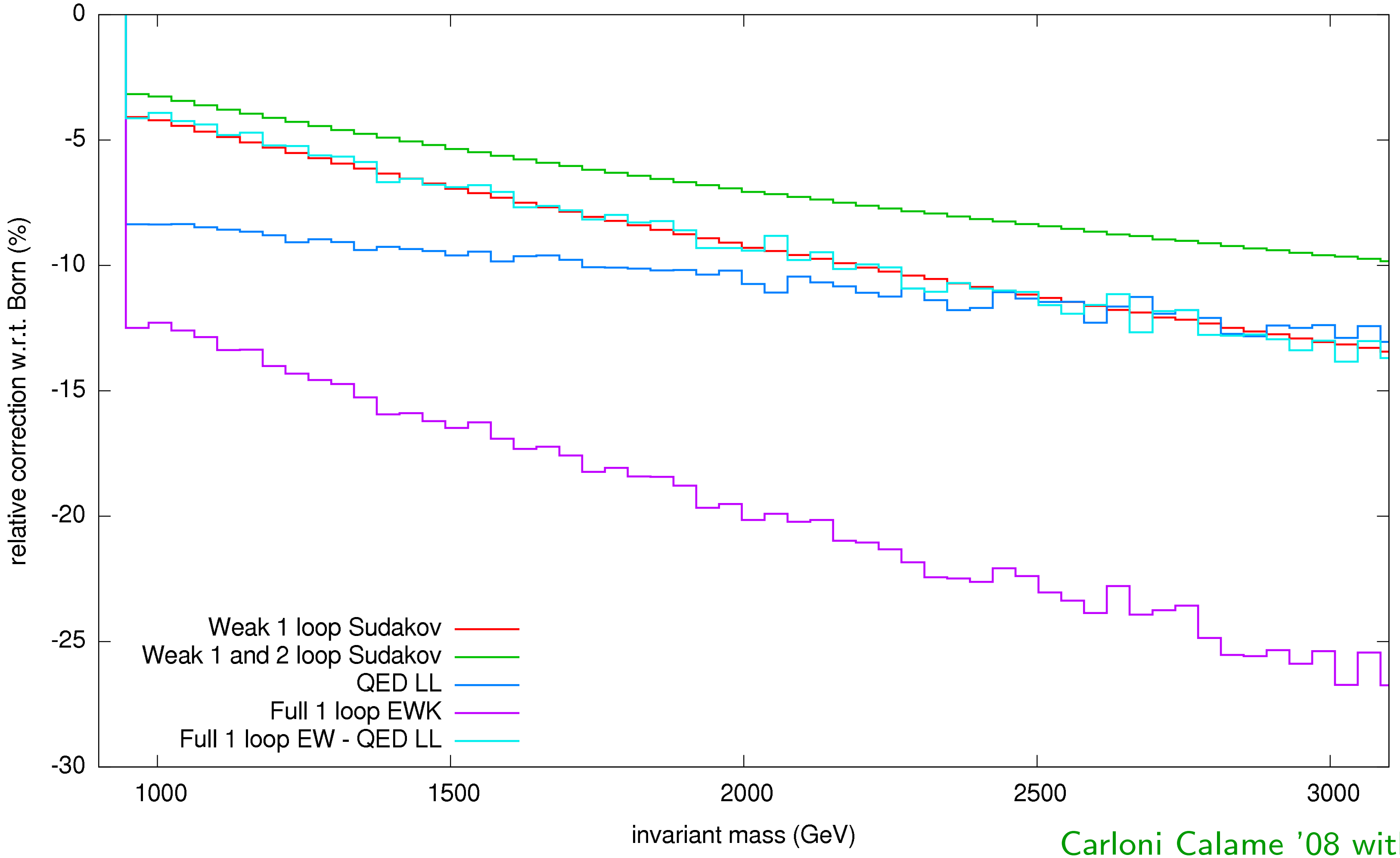
$\Rightarrow$ enhanced by large Sudakov logarithms

$$\text{per loop:} \quad \ln^2\left(\frac{s}{M_{\mathrm{W}}^2}\right) \sim 25 \quad \text{at } \sqrt{s} \sim 1\,\text{TeV}$$

- $M_{W,Z} \neq 0 \rightarrow$ exclusive observables possible: only virtual W's and Z's
  ($\neq$ QED, QCD where singular logs cancel between virtual and real corrections)
- large logs even in inclusive observables (Bloch–Nordsieck violations)

EW corrections at the LHC

Drell–Yan $pp \to \mu^+\mu^- + X$**:** (electro)weak 1-loop & 2-loop corrections



Carloni Calame '08 with HORACE
and Sudakov results from
B.J., Kühn, Penin, Smirnov '05

$\Rightarrow$ Sudakov approximation very good at high energies

$\Rightarrow$ 2-loop effects $\sim \mathcal{O}(\%)$

General form of EW corrections for $s \gg M_W^2$

$$\left[L = \ln\left(\frac{s}{M_W^2}\right)\right]$$

↪ LL (leading logarithmic), NLL (next-to-leading logarithmic), ... terms:

1 loop: $\alpha \left[C_1^{\mathrm{LL}} L^2 + C_1^{\mathrm{NLL}} L + C_1^{\mathrm{N^2LL}} \right] + \mathcal{O}\left(\frac{M_W^2}{s}\right)$

$$C_1^{\mathrm{LL}} L^2 \to -17\,\%, \quad C_1^{\mathrm{NLL}} L \to +12\,\%, \quad C_1^{\mathrm{N^2LL}} \to -3\,\%$$

2 loops: $\alpha^2 \left[C_2^{\mathrm{LL}} L^4 + C_2^{\mathrm{NLL}} L^3 + C_2^{\mathrm{N^2LL}} L^2 + C_2^{\mathrm{N^3LL}} L + C_2^{\mathrm{N^4LL}} \right] + \mathcal{O}\left(\frac{M_W^2}{s}\right)$

$$C_2^{\mathrm{LL}} L^4 \to +1.7\,\%, \quad C_2^{\mathrm{NLL}} L^3 \to -1.8\,\%, \quad C_2^{\mathrm{N^2LL}} L^2 \to +1.2\,\%, \quad C_2^{\mathrm{N^3LL}} L \to -0.3\,\%$$

[$\sigma(u\bar{u} \to d\bar{d})$ @ $\sqrt{s} = 1$ TeV, B.J., Kühn, Penin, Smirnov '05]

For theoretical predictions with accuracy $\sim 1\,\%$:

⇒ 2-loop corrections important

⇒ LL approximation not sufficient

With massless photons: log $\rightsquigarrow 1/\epsilon$ in $D = 4 - 2\epsilon$ dimensions

Virtual 2-loop EW corrections

Resummation of 1-loop results:

- LL & NLL for arbitrary processes $(M_\mathrm{Z} = M_\mathrm{W})$ — Fadin, Lipatov, Martin, Melles '99; Melles '00, '01
- N^{2}LL for $f\bar{f} \to f'\bar{f}'$ $(m_f = 0,\, M_\mathrm{Z} = M_\mathrm{W})$ — Kühn, Penin, Smirnov '99, '00; Kühn, Moch, Penin, Smirnov '01
- N^{2}LL for $e^+e^- \to \mathrm{W}^+\mathrm{W}^-$ — Kühn, Metzler, Penin '07
- SCET method — Chiu, Golf, Kelley, Manohar '07

$\longrightarrow$ apply evolution equations to spontaneously broken Standard Model

$\hookrightarrow$ split theory into symmetric SU(2)×U(1) regime & QED regime

Diagrammatic 2-loop calculations $\rightarrow$ check & extend resummation results:

- LL & angular-dependent NLLs for arbitrary processes — Melles '00; Hori, Kawamura, Kodaira '00; Beenakker, Werthenbach '00, '01; Denner, Melles, Pozzorini '03
- NLL for fermionic processes $(m_f = 0,\, M_\mathrm{Z} \neq M_\mathrm{W})$ — Pozzorini '04; Denner, B.J., Pozzorini '06
- N^{3}LL for fermionic form factor $(m_f = 0,\, M_\mathrm{Z} = M_\mathrm{W})$
  $\hookrightarrow$ N^{3}LL for $f\bar{f} \to f'\bar{f}'$ $(m_f = 0,\, M_\mathrm{Z} \approx M_\mathrm{W})$ via evolution equations
  B.J., Kühn, Moch '03; B.J., Kühn, Penin, Smirnov '04, '05

II Two-loop next-to-leading logarithmic corrections

Goal: virtual 2-loop EW corrections for arbitrary processes in NLL accuracy
$\hookrightarrow$ start with processes involving massless & massive external fermions

Parameters:

- different large kinematical invariants $r_{ij} = (p_i + p_j)^2 \sim Q^2 \gg M_\mathrm{W}^2$
- different heavy particle masses $M_\mathrm{W}^2 \sim M_\mathrm{Z}^2 \sim m_\mathrm{t}^2 \sim M_\mathrm{Higgs}^2$
- massive top quark, other fermions massless

$\Rightarrow$ logs $L = \ln\left(\frac{Q^2}{M_\mathrm{W}^2}\right)$ and $\frac{1}{\epsilon}$ poles (from virtual photons)

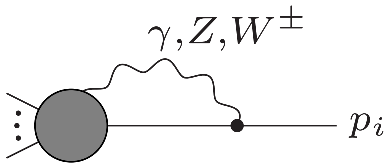
1 loop: LL $\to \epsilon^{-2},\ L\epsilon^{-1},\ L^2,\ L^3\epsilon,\ L^4\epsilon^2$; NLL $\to \epsilon^{-1},\ L,\ L^2\epsilon,\ L^3\epsilon^2$

2 loops: LL $\to \epsilon^{-4},\ L\epsilon^{-3},\ L^2\epsilon^{-2},\ L^3\epsilon^{-1},\ L^4$; NLL $\to \epsilon^{-3},\ L\epsilon^{-2},\ L^2\epsilon^{-1},\ L^3$

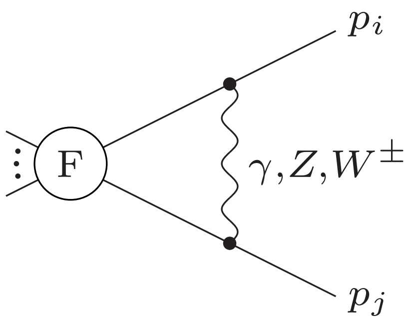
$\Rightarrow$ NLL coefficients involve small logs $\ln\left(\frac{-r_{ij}}{Q^2}\right)$ and $\ln\left(\frac{M_\mathrm{Z}^2, m_\mathrm{t}^2}{M_\mathrm{W}^2}\right)$

Extraction of NLL logs at 1 loop

Logs originate from mass singularities
in collinear/soft regions:
(+ UV logs)

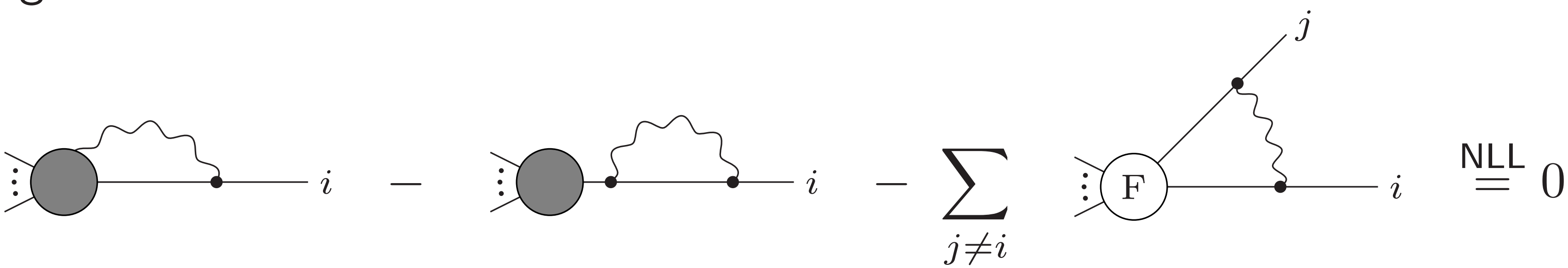


Isolate **factorizable contributions:**



$\hookrightarrow$ separate loop integral from Born diagram (F) via soft–collinear approximation

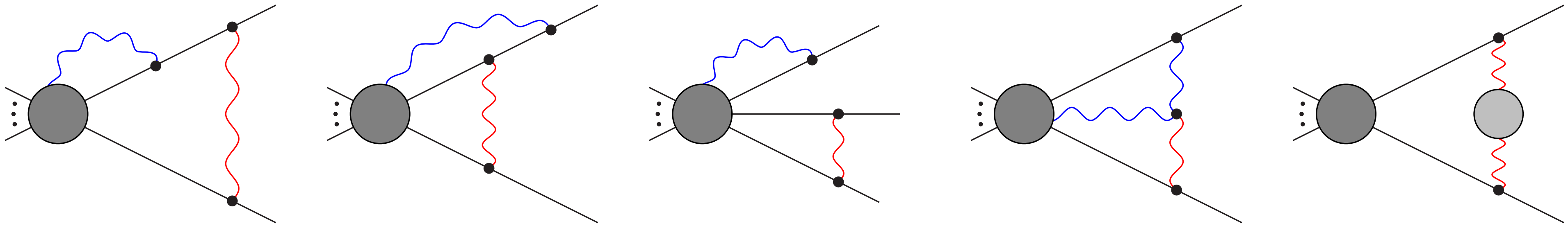
Remaining non-factorizable contributions: collinear Ward identities Denner, Pozzorini '01



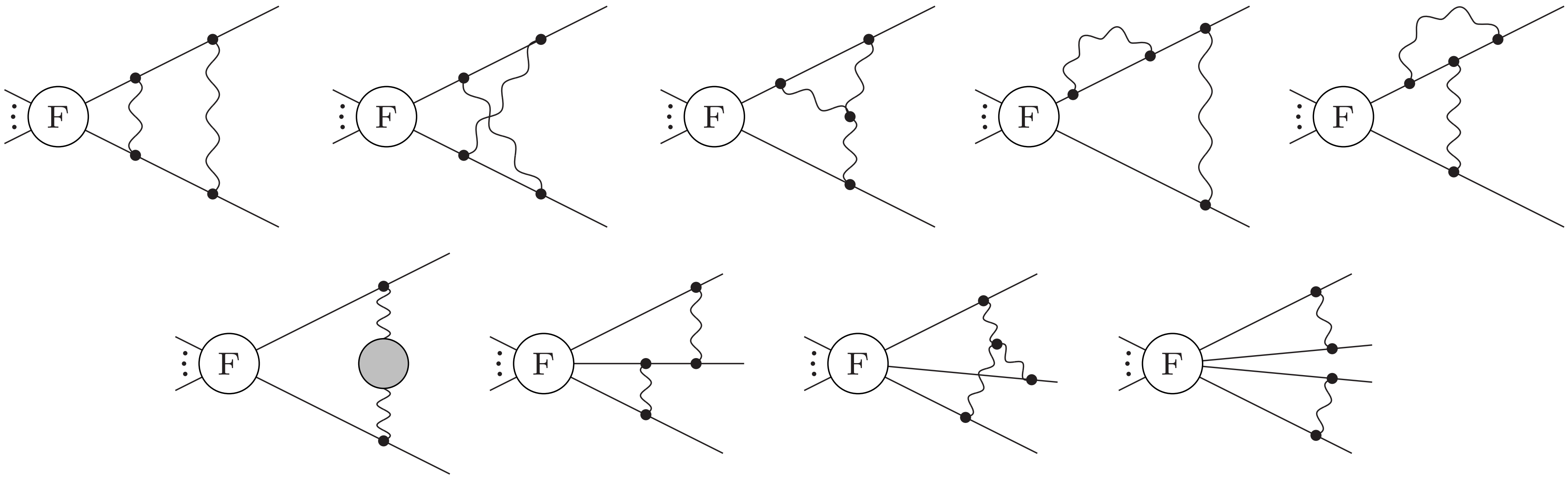
Factorizable contributions contain <u>all</u> soft & collinear NLL mass singularities.

Extraction of NLL logs at 2 loops

↪ contributions: soft×soft and soft×collinear (without Yukawa contributions):



Factorizable contributions:



- calculated with soft–collinear approximation (and projection techniques)
- non-factorizable contributions vanish

Denner, B.J., Pozzorini '06

Treatment of ultraviolet (UV) singularities

UV $1/\epsilon$ poles in subdiagrams with scale μ^2_{loop} & renormalization at scale μ^2_{R}:

$$\underbrace{\frac{1}{\epsilon}\left(\frac{Q^2}{\mu^2_{\mathsf{loop}}}\right)^{\epsilon}}_{\text{bare diagrams}} - \underbrace{\frac{1}{\epsilon}\left(\frac{Q^2}{\mu^2_{\mathsf{R}}}\right)^{\epsilon}}_{\text{counterterms}} = \ln\left(\frac{\mu^2_{\mathsf{R}}}{\mu^2_{\mathsf{loop}}}\right) + \mathcal{O}(\epsilon) \quad \Rightarrow \quad \text{possible NLL contribution}$$

Perform **minimal UV subtraction** in UV-singular (sub)diagrams and counterterms:

$$\underbrace{\frac{1}{\epsilon}\left[\left(\frac{Q^2}{\mu^2_{\mathsf{loop}}}\right)^{\epsilon} - 1\right]}_{\text{bare diagrams}} - \underbrace{\frac{1}{\epsilon}\left[\left(\frac{Q^2}{\mu^2_{\mathsf{R}}}\right)^{\epsilon} - 1\right]}_{\text{counterterms}}$$

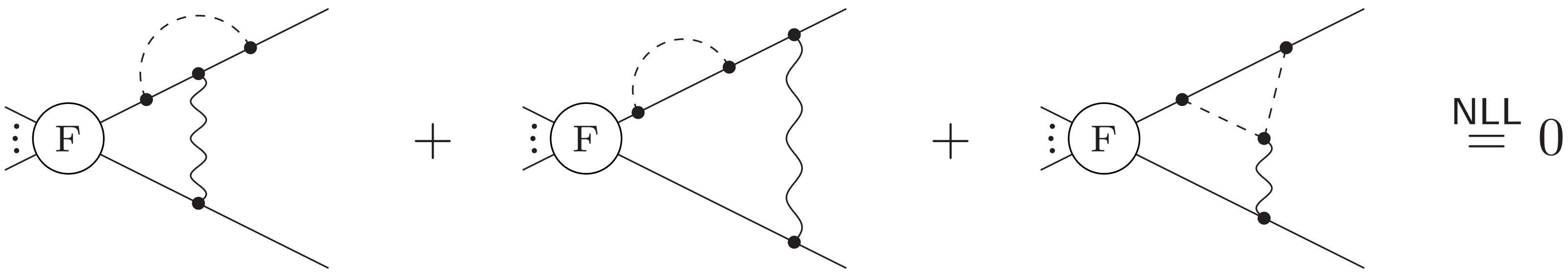
Advantages:

- no UV NLL terms from hard subdiagrams ($\mu^2_{\mathsf{loop}} \sim Q^2$)
  $\hookrightarrow$ no UV contributions from internal parts of tree subdiagrams
- can use soft–collinear approximation (not valid in UV regime!) also for hard UV-singular subdiagrams

Yukawa contributions

Massive fermions → Yukawa couplings to scalars (Higgs, Goldstone bosons)

- many Yukawa contributions are suppressed (soft/collinear limit, $M_W^2/Q^2 \to 0$)
- only <u>three</u> non-suppressed factorizable diagrams:



Sum vanishes due to gauge invariance of Yukawa interaction
↪ NLL Yukawa contributions only in wave-function renormalization

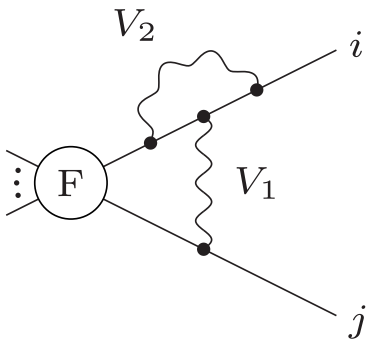
III Results for massless and massive fermionic processes

Factorizable contributions

loop integrals calculated with two independent methods:

- automatized algorithm based on sector decomposition — Denner, Pozzorini '04
- combination of expansion by regions & Mellin–Barnes representations — B.J., Smirnov '06 & refs. therein

Example:



$$= \underbrace{\sum_{V_1,V_2=\gamma,Z,W^\pm}}_{\text{sum over gauge bosons}} \overbrace{D(M_{V_1},M_{V_2};p_i,p_j)}^{\text{scalar 2-loop integral}} \underbrace{I_i^{V_2} I_i^{V_1} I_i^{\bar{V}_2} I_j^{\bar{V}_1}}_{\text{isospin matrices @ external legs}} \quad \mathcal{M}_0$$

Born amplitude factorized ↑ ($\mathcal{M}_0$)

All relevant combinations of $\left\{\begin{matrix}\text{massless}\\ \text{massive}\end{matrix}\right\}$ $\left\{\begin{matrix}\text{external}\\ \text{internal}\end{matrix}\right\}$ fermions evaluated explicitly!

Result for amplitude of fermionic processes $f_1 f_2 \to f_3 \cdots f_n$ in $\mathcal{O}(\alpha^2)$

$$\mathcal{M} \overset{\text{NLL}}{=} \underbrace{\exp\big(\Delta F^{\text{em}}\big)}_{\substack{\text{electromagnetic}\\ M_\gamma = 0}} \times \underbrace{\exp\big(F^{\text{sew}}\big)}_{\substack{\text{symmetric-electroweak}\\ M_\gamma = M_Z = M_W}} \times \underbrace{\big(1 + \Delta F^{Z}\big)}_{\substack{\text{corrections}\\ \text{from } M_Z \neq M_W}} \times \underbrace{\mathcal{M}_0}_{\text{Born}}$$

- universal result: F^{sew}, ΔF^{em}, ΔF^{Z} depend only on external quantum numbers
- electromagnetic singularities (in ΔF^{em}) factorized $\to$ separable

Symmetric-electroweak terms: independent of fermion masses

$$F^{\text{sew}} = \frac{1}{2} \sum_{i=1}^{n} \Bigg\{ -\frac{\alpha}{4\pi} \Bigg[\sum_{j \neq i} \sum_{V=\gamma,Z,W^\pm} I_i^{\bar{V}} I_j^{V}\, I_{ij}(\epsilon, M_W) + \overbrace{\frac{z_i^{\text{Yuk}}\, m_t^2}{4 s_w^2 M_W^2} \left(L + \frac{1}{2} L^2 \epsilon + \frac{1}{6} L^3 \epsilon^2 \right)}^{\text{Yukawa contribution}} \Bigg]$$
$$+ \left(\frac{\alpha}{4\pi} \right)^2 \left[\frac{b_1^{(1)}}{c_w^2} \left(\frac{Y_i}{2} \right)^2 + \frac{b_2^{(1)}}{s_w^2} C_i^{w} \right] J_{ii}(\epsilon, M_W, \mu_R^2) \Bigg\},$$

$$I_{ij}(\epsilon, M_W) \overset{\text{NLL}}{=} -L^2 - \frac{2}{3} L^3 \epsilon - \frac{1}{4} L^4 \epsilon^2 + \left[3 - 2 \ln\left(\frac{-r_{ij}}{Q^2} \right) \right] \left(L + \frac{1}{2} L^2 \epsilon + \frac{1}{6} L^3 \epsilon^2 \right) + \mathcal{O}(\epsilon^3),$$

$$J_{ij}(\epsilon, M_W, \mu_R^2) = \frac{1}{\epsilon} \left[I_{ij}(2\epsilon, M_W) - \left(\frac{Q^2}{\mu_R^2} \right)^{\epsilon} I_{ij}(\epsilon, M_W) \right]$$

$$\mathcal{M} \stackrel{\text{NLL}}{=} \exp\big(\Delta F^{\text{em}}\big) \times \exp\big(F^{\text{sew}}\big) \times \big(1 + \Delta F^{\text{Z}}\big) \times \mathcal{M}_0$$

Electromagnetic terms: QED$(M_\gamma = 0)$ − QED$(M_\gamma = M_\text{W})$ $[\mu_\text{R}^2 = M_\text{W}^2]$

$$\Delta F^{\text{em}} = \frac{1}{2} \sum_{i=1}^{n} \Bigg\{ -\frac{\alpha}{4\pi} \sum_{j\neq i} Q_i Q_j \Big[I_{ij}(\epsilon, 0) - I_{ij}(\epsilon, M_\text{W}) \Big] + \left(\frac{\alpha}{4\pi}\right)^2 b^{(1)}_{\text{QED}} Q_i^2 \Big[J_{ii}(\epsilon, 0, M_\text{W}^2) - J_{ii}(\epsilon, M_\text{W}, M_\text{W}^2) \Big] \Bigg\},$$

$$I_{ij}(\epsilon, 0) \stackrel{\text{NLL}}{=} -\left[3 - 2\ln\left(\frac{-r_{ij}}{Q^2}\right)\right]\epsilon^{-1} + \Bigg\{ \overbrace{-\delta_{i,0}\,\epsilon^{-2} + \delta_{i,\text{t}}}^{\text{dependence on fermion mass } m_i} \bigg[L\epsilon^{-1} + \frac{1}{2}L^2 + \frac{1}{6}L^3\epsilon + \frac{1}{24}L^4\epsilon^2 + \left(\frac{1}{2} - \ln\left(\frac{m_i^2}{M_\text{W}^2}\right)\right)\left(\epsilon^{-1} + L + \frac{1}{2}L^2\epsilon + \frac{1}{6}L^3\epsilon^2\right) \bigg] + (i \to j) \Bigg\} + \mathcal{O}(\epsilon^3),$$

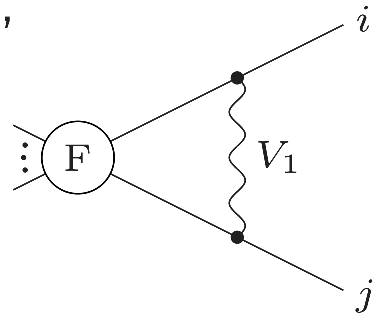
$$J_{ij}(\epsilon, 0, \mu_\text{R}^2) = \frac{1}{\epsilon}\left[I_{ij}(2\epsilon, 0) - \left(\frac{Q^2}{\mu_\text{R}^2}\right)^{\epsilon} I_{ij}(\epsilon, 0) \right]$$

Terms from $M_\text{Z} \neq M_\text{W}$:

$$\Delta F^{\text{Z}} \stackrel{\text{NLL}}{=} \frac{\alpha}{4\pi} \sum_{i=1}^{n} (I_i^{\text{Z}})^2 \overbrace{\ln\left(\frac{M_\text{Z}^2}{M_\text{W}^2}\right)\left(L + L^2\epsilon + \frac{1}{2}L^3\epsilon^2\right)}^{= I_{ii}(\epsilon, M_\text{Z}) - I_{ii}(\epsilon, M_\text{W})} + \mathcal{O}(\epsilon^3)$$

IV Structure of the result to all orders in ϵ

Expansion by regions: look at contributions from individual regions,
e.g. in 1-loop diagram (with minimal UV subtraction):



hard region: $-2\left(\epsilon^{-2}+2\epsilon^{-1}\right)\left(\frac{Q^2}{-r_{ij}}\right)^{\epsilon}$

collinear regions:

- $m_1 = 0 \Rightarrow \left(\epsilon^{-2}+2\epsilon^{-1}\right)\left[\left(\frac{Q^2}{m_i^2}\right)^{\epsilon}+\left(\frac{Q^2}{m_j^2}\right)^{\epsilon}\right]$
- $m_1 \neq 0 \Rightarrow 2\left[\epsilon^{-2}-\underbrace{\ln\left(\frac{-r_{ij}}{m_1^2}\right)}\epsilon^{-1}+2\epsilon^{-1}\right]\left(\frac{Q^2}{m_1^2}\right)^{\epsilon}$

finite remainder from singularity cancelled between i-/j-collinear regions

Each region depends on mass parameters via **one unique power** of $(Q^2/m^2)^{\epsilon}$.

$\hookrightarrow$ Logs $\ln(Q^2/m^2)$ are generated by poles ϵ^{-n} in prefactor.

$\hookrightarrow$ Additional logarithms arise from singularities cancelled between collinear regions.

$\hookrightarrow$ $\mathcal{O}(\epsilon^0)$ in prefactor is beyond NLL accuracy.

$\rightsquigarrow$ In NLL accuracy this representation is valid **to all orders in ϵ**!

NLL result to all orders in ϵ

$$\mathcal{M} \overset{\text{NLL}}{=} \exp\left(\Delta F^{\text{em}}\right) \times \exp\left(F^{\text{sew}}\right) \times \left(1 + \Delta F^{\text{Z}}\right) \times \mathcal{M}_0$$

Every part of the result is known to all orders in ϵ. Most important ingredient:

$$I_{ij}(\epsilon, m_1) \overset{\text{NLL}}{=} -\left(2\epsilon^{-2} + 3\epsilon^{-1}\right) z_{ij}^{-\epsilon} + \left\{\left[2\epsilon^{-2} - 2L\epsilon^{-1} + (3 - 2l_{ij} + 2l_1)\,\epsilon^{-1}\right] z_1^{-\epsilon} + \delta_{1,\gamma}\left(\epsilon^{-2} + \frac{1}{2}\epsilon^{-1}\right)\left(z_i^{-\epsilon} + z_j^{-\epsilon}\right)\right\} Z^{\epsilon}$$

with

$$Z = \frac{Q^2}{M_{\text{W}}^2},\ L = \ln Z;\ z_{ij} = \frac{-r_{ij}}{Q^2},\ l_{ij} = \ln z_{ij};\ z_a = \frac{m_a^2}{M_{\text{W}}^2},\ l_a = \ln z_a,\ a = 1, 2, \ldots, i, j, \ldots$$

$$z_a^{n\epsilon} \equiv 0 \text{ if } m_a = 0,\ n \neq 0; \qquad \delta_{a,\gamma} = \begin{cases} 1, & m_a = 0 \\ 0, & m_a \sim M_{\text{W}} \end{cases}$$

Exponentiation

1-loop $\rightsquigarrow I_{ij}(\epsilon, m_1)$: Z^0 (hard), Z^{ϵ} (collinear)

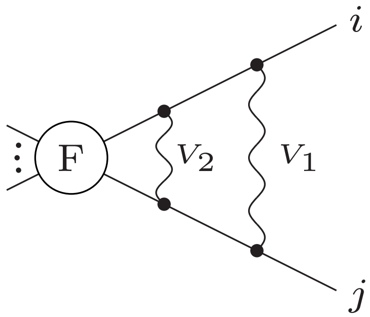
2-loop $\rightsquigarrow I_{ij}(\epsilon, m_1) \times I_{kl}(\epsilon, m_2),\ I_{ij}(2\epsilon, m_1),\ Z^{\epsilon} \times I_{ij}(\epsilon, m_1)$:

Z^0 (hard–hard), Z^{ϵ} (hard–collinear), $Z^{2\epsilon}$ (collinear–collinear)

2-loop contributions to all orders in ϵ

2-loop result involves Z^0 (hard–hard), Z^ϵ (hard–collinear), $Z^{2\epsilon}$ (collinear–collinear)

But:

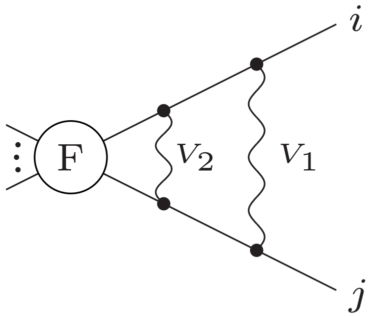


$$\begin{aligned}
&\hookrightarrow \left(\epsilon^{-4}+4\epsilon^{-3}\right) z_{ij}^{-2\epsilon} \\
&+ \left\{4\left[\epsilon^{-4}+L\epsilon^{-3}+(l_{ij}-l_1)\,\epsilon^{-3}+2L\epsilon^{-2}\right] z_1^{-\epsilon} - \frac{2}{3}\delta_{1,\gamma}\left(\epsilon^{-4}+4\epsilon^{-3}\right)\left(z_i^{-\epsilon}+z_j^{-\epsilon}\right)\right\} z_{ij}^{-\epsilon} Z^{\epsilon} \\
&+ \Big\{-\left[5\epsilon^{-4}-2L\epsilon^{-3}+2\,(2-l_{ij}+l_1)\,\epsilon^{-3}\right] z_1^{-2\epsilon} \\
&\quad + \delta_{1,\gamma}\left[-\left(\epsilon^{-4}-2L\epsilon^{-3}+2\,(2-l_{ij}+l_2)\,\epsilon^{-3}\right) z_2^{-2\epsilon} + \left(\epsilon^{-4}+2\epsilon^{-3}\right) z_2^{-\epsilon}\left(z_i^{-\epsilon}+z_j^{-\epsilon}\right)\right]\Big\} Z^{2\epsilon} \\
&+ \delta_{1,\gamma}\left\{-\frac{1}{3}\left(\epsilon^{-4}-2\epsilon^{-3}\right) z_2^{-3\epsilon}\left(z_i^{-\epsilon}+z_j^{-\epsilon}\right) + \frac{1}{6}\delta_{2,\gamma}\left(\epsilon^{-4}+4\epsilon^{-3}\right)\left(z_i^{-\epsilon}z_j^{-3\epsilon}+z_j^{-\epsilon}z_i^{-3\epsilon}\right)\right\} z_{ij}^{2\epsilon} Z^{4\epsilon}
\end{aligned}$$

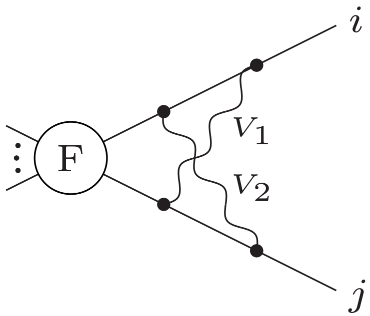
new contribution: $Z^{4\epsilon}$ (ultracollinear–collinear), not present in 2-loop result?!

$\hookrightarrow$ non-trivial cancellation of all $Z^{4\epsilon}$ terms in total amplitude!

Cancellation of $Z^{4\epsilon}$ terms in 2-loop result

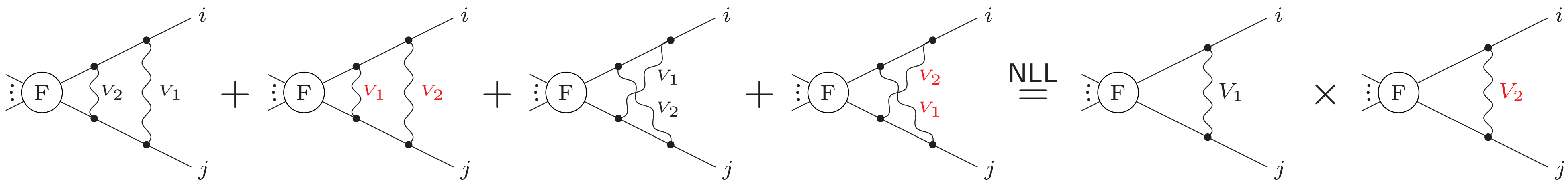


$$\rightsquigarrow \quad \delta_{1,\gamma}\left\{-\frac{1}{3}\left(\epsilon^{-4}-2\epsilon^{-3}\right) z_2^{-3\epsilon}\left(z_i^{-\epsilon}+z_j^{-\epsilon}\right) + \frac{1}{6}\delta_{2,\gamma}\left(\epsilon^{-4}+4\epsilon^{-3}\right)\left(z_i^{-\epsilon}z_j^{-3\epsilon}+z_j^{-\epsilon}z_i^{-3\epsilon}\right)\right\} z_{ij}^{2\epsilon}\, {\color{red}Z^{4\epsilon}}$$



$$\rightsquigarrow \quad \left\{\frac{1}{3}\left(\epsilon^{-4}-2\epsilon^{-3}\right)\left(\delta_{1,\gamma}z_2^{-3\epsilon}z_i^{-\epsilon}+\delta_{2,\gamma}z_1^{-3\epsilon}z_j^{-\epsilon}\right) - \frac{1}{6}\delta_{1,\gamma}\delta_{2,\gamma}\left(\epsilon^{-4}+4\epsilon^{-3}\right)\left(z_i^{-\epsilon}z_j^{-3\epsilon}+z_j^{-\epsilon}z_i^{-3\epsilon}\right)\right\} z_{ij}^{2\epsilon}\, {\color{red}Z^{4\epsilon}}$$

$\Rightarrow$ ${\color{red}Z^{4\epsilon}}$ terms cancel in combination of scalar loop integrals:



$\hookrightarrow$ this relation (and others) checked to all orders in ϵ ✓

V Summary & outlook

Massless and massive fermionic processes $f_1 f_2 \longrightarrow f_3 \cdots f_n$

with $(p_i + p_j)^2 \gg M_\mathrm{W}^2$ and different masses $M_\mathrm{W}^2 \sim M_\mathrm{Z}^2 \sim m_\mathrm{t}^2 \sim M_\mathrm{Higgs}^2$:

- 2-loop EW NLL corrections in $D = 4 - 2\epsilon$ dimensions

  [$m_f = 0$: Denner, B.J., Pozzorini, Nucl. Phys. B 761 (2007) 1]
- loop integrals calculated with two independent methods
- Yukawa contributions only in wave-function renormalization
- universal correction factors, electromagnetic singularities separable
- applicable for $e^+ e^- \longrightarrow f \bar{f}$, Drell–Yan, ...
- NLL result available to all orders in ϵ $\longrightarrow$ structure with powers of $(Q^2/M_\mathrm{W}^2)^\epsilon$

  $\hookrightarrow$ non-trivial cancellations ensure exponentiation of 1-loop result

Outlook: arbitrary processes

- generalize method for external gauge bosons & scalars (Higgs)
- calculate relevant loop integrals
- goal: process-independent 2-loop NLL corrections