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The Non-Abelian Form Factor

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I Why logarithmic 2-loop calculations in EW theory?

Electroweak (EW) precision physics

- experimentally measured by now at energy scales up to $\sim M_{W,Z}$
- up-coming generation of accelerators (LHC, ILC) $\rightarrow$ TeV region

Electroweak radiative corrections at high energies $\sqrt{s} \sim \text{TeV} \gg M_{W,Z}$

Fadin et al. '00; Kühn et al. '00, '01; Denner et al. '01, '03, '04; Pozzorini '04; B.F. et al. '03, '04; ...

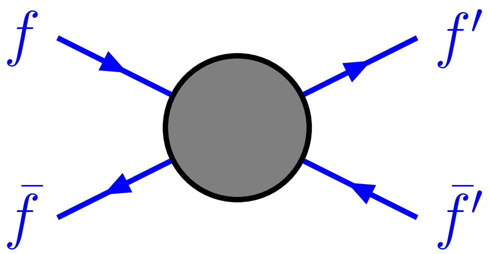
large negative corrections in *exclusive* cross sections

electroweak corrections dominated by Sudakov logarithms $\alpha^n \ln^{2n}(s/M_W^2)$

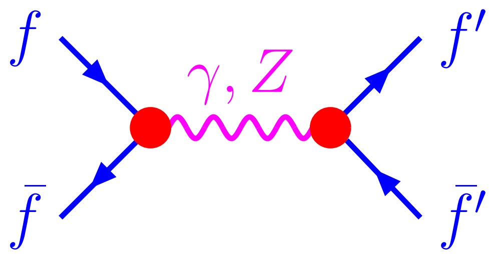
- 1-loop corrections $\gtrsim 10\%$
- 2-loop corrections $\sim 1\%$ $\rightarrow$ need to be under control

Problem: loop calculations with massive particles (W, Z) difficult!

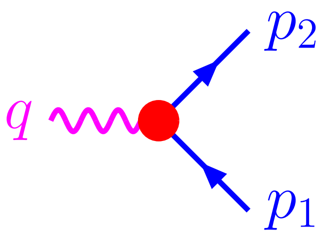
Important class of processes: 4-fermion scattering



$$A = \frac{ig^2}{s} F^2 \tilde{A}$$



Form factor F of vector current:



$$= F \cdot \bar{u}(p_2)\, \gamma^\mu\, u(p_1) + \underbrace{F' \cdot \bar{u}(p_2)\, \sigma^{\mu\nu}\, u(p_1)\, q_\nu}_{\to 0,\ m_f \to 0}$$

High energy behaviour $|s| \sim |t| \sim |u| \gg M_{W,Z}^2$

references: see Kühn et al. '01

- all *collinear* logarithms of amplitude $A \rightsquigarrow$ form factors F^2
- *reduced amplitude* $\tilde{A} \to$ only *soft* logarithms
- $\tilde{A}$ satisfies an *evolution equation* (known from massless QCD calculations):

$$\frac{\partial \tilde{A}}{\partial \ln s} = \chi\big(\alpha(s)\big)\, \tilde{A}\,, \quad \chi = \text{matrix of soft anomalous dimensions}$$

$\Rightarrow$ still needed for 2-loop logarithms in A: form factor F

Simplified models

Standard Model

$SU(2)_L \times U(1)_Y$

mixing $(\sin\theta_W \neq 0)$

$M_W \neq M_Z,\ M_\gamma = 0$

$\longrightarrow$

$SU(2)_M \times U(1)_\lambda$ model

no mixing $(\sin\theta_W \approx 0)$

$\hookrightarrow M \equiv M_W \approx M_Z$

mass gap $M \gg \lambda \to 0$

$\longrightarrow$

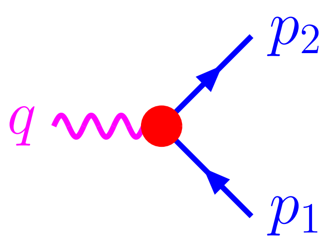
massive SU(2) model

only weak int. without γ

1 mass $M \neq 0$

High energy behaviour of the form factor

$\hookrightarrow$ Sudakov limit:



$$= F(Q^2)\cdot \bar{u}(p_2)\,\gamma^\mu\,u(p_1)$$

- momentum transfer $-q^2 \equiv Q^2 \gg M^2 \equiv M^2_{W,Z}$
- neglect fermion masses
- *logarithmic approximation*: neglect terms $\propto M^2/Q^2$

  $\hookrightarrow$ good approximation for 2-loop n_f contribution B.F., Kühn, Moch '03

II The massive SU(2) form factor

Form factor in perturbation theory: $F = 1 + \alpha\, F_1 + \alpha^2\, F_2 + \ldots$

Evolution equation in logarithmic approximation: Sen '81; Collins '89; Korchemsky '89; ...

$$\frac{\partial F(Q^2)}{\partial \ln Q^2} = \left[\int_{M^2}^{Q^2} \frac{dx}{x} \gamma\big(\alpha(x)\big) + \zeta\big(\alpha(Q^2)\big) + \xi\big(\alpha(M^2)\big) \right] F(Q^2)$$

solution (schematically):

$$F = \big(1 + \alpha \cdot \text{const} + \alpha^2 \cdot \text{const} + \ldots\big) \exp\Big(\alpha\,(\ln^2 + \ln) + \alpha^2\,(\ln^3 + \ln^2 + \ln) + \ldots\Big)$$

$$\leftrightarrow 1 + \alpha\,(\ln^2 + \ln + \text{const}) + \alpha^2\,(\ln^4 + \ln^3 + \ln^2 + \ln + \text{const}) + \ldots$$

Combination of loop calculations & evolution equation

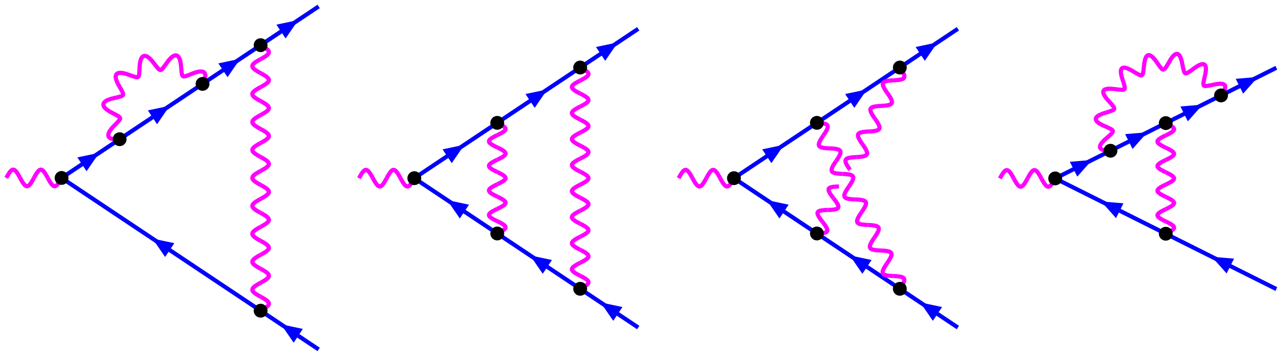
anomalous dimensions γ, ζ, ξ from 1-loop calculation & <u>massless</u> 2-loop result

$\Rightarrow$ obtain NNLL approximation of F_2: $\alpha^2\,(\ln^4 + \ln^3 + \ln^2)$ Kühn, Moch, Penin, Smirnov '01

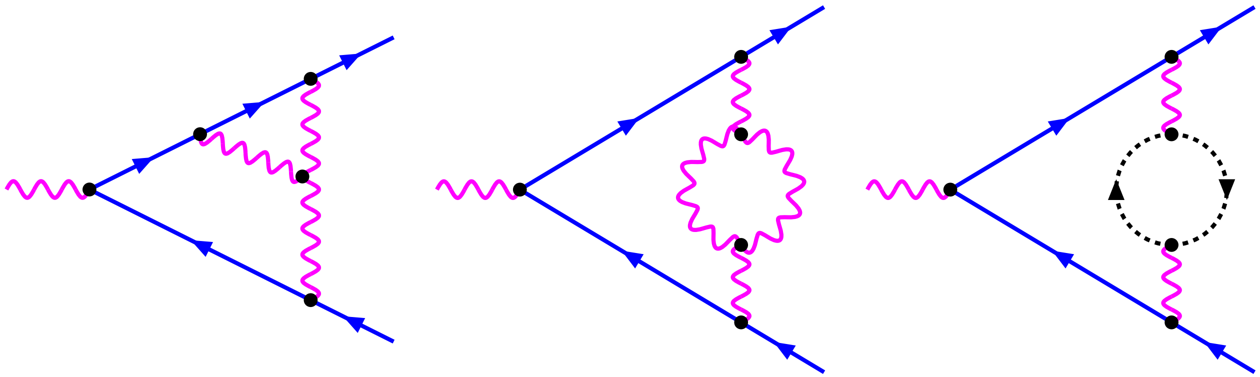
Massive SU(2) form factor in 2-loop approximation: contributions & diagrams

2-loop vertex diagrams (massless fermions, massive bosons):

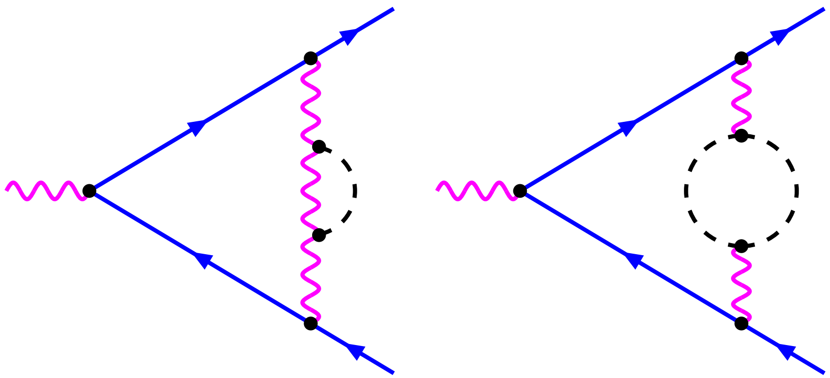
Abelian (C_F^2):



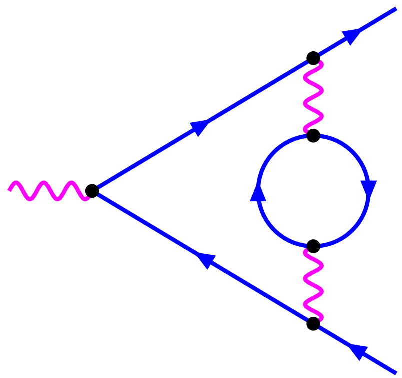
non-Abelian ($C_F C_A$): last 2 +



Higgs:



fermion ($C_F T_F n_f$):



$+$ 1-loop$\times$1-loop corrections $+$ renormalization

Size of the logarithmic contributions

2-loop form factor F_2 at $Q = 1$ TeV (in 1/1000):

Abelian (C_F^2):	$+0.3\,\ln^4$	$-1.7\,\ln^3$	$+8.2\,\ln^2$	$-11\,\ln$	$+15$
	+1.6	−2.0	+1.9	−0.5	+0.1
non-Abelian ($C_F C_A$):		$+1.8\,\ln^3$	$-14\,\ln^2$	$+46\,\ln$	$-\ldots$
		+2.1	−3.3	+2.1	
Higgs:		$-0.04\,\ln^3$	$+0.5\,\ln^2$	$-2.3\,\ln$	$+\ldots$
		−0.04	+0.1	−0.1	
fermionic ($C_F T_F n_f$):		$-0.5\,\ln^3$	$+4.8\,\ln^2$	$-13\,\ln$	$+21$
		−0.6	+1.1	−0.6	+0.2

$\ln^{4,3,2}$: Kühn, Moch, Penin, Smirnov '01

$\ln^{1,0}$: B.F., Kühn, Moch '03; B.F., Kühn, Penin, Smirnov '04

$\longrightarrow$ growing coefficients with alternating signs

$\Rightarrow$ cancellations between logarithmic terms

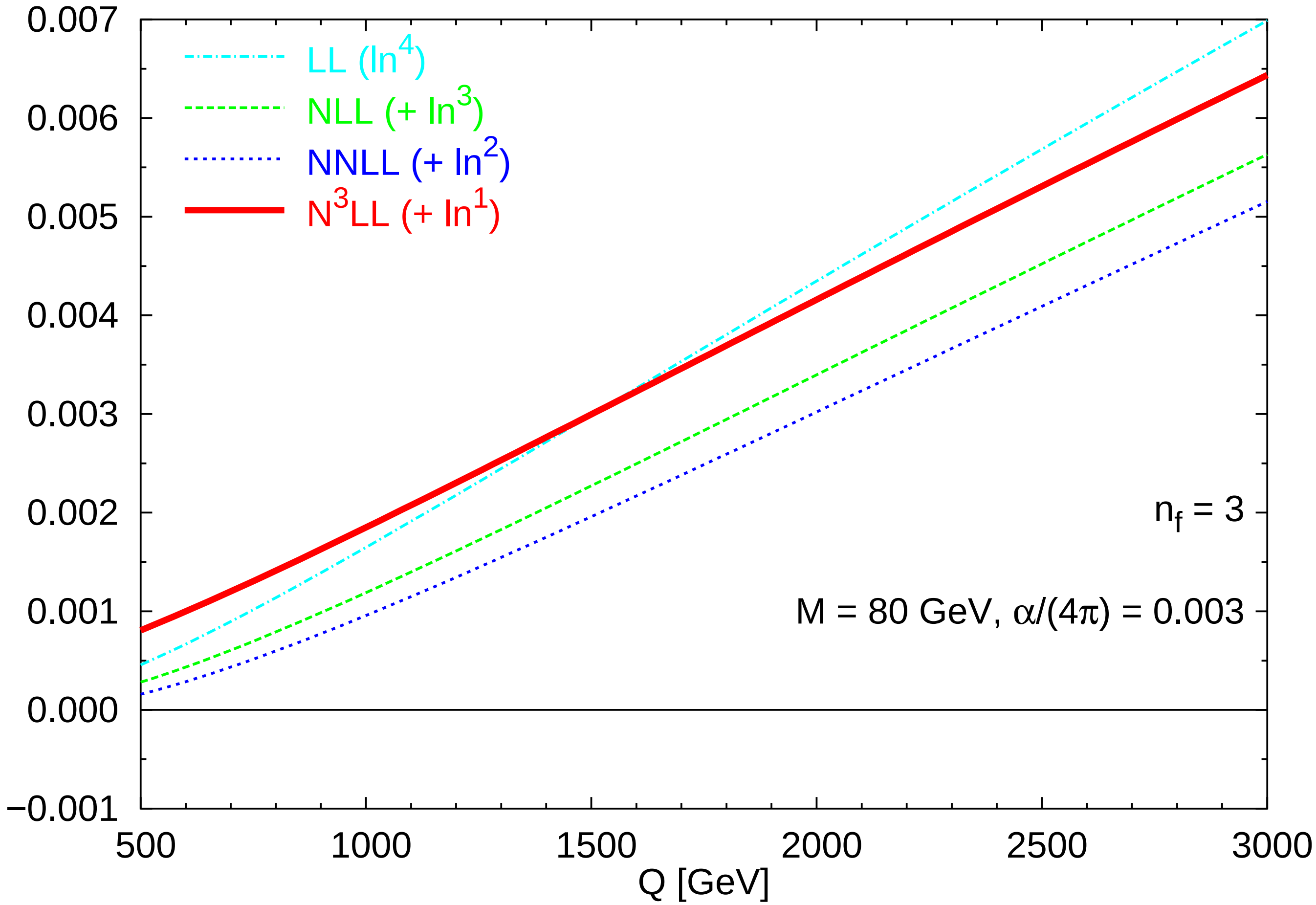
$\hookrightarrow$ **NNLL approximation is not enough!**

Abelian & fermionic contribution: $\ln^1$ small, $\ln^0$ negligible

$\Rightarrow$ **N^{3}LL approximation** including $\ln^1$ is sufficient (non-Abelian $\ln^0$ more difficult)

Massive SU(2) form factor in 2-loop approximation: result

$$\alpha^2 F_2 = \left(\frac{\alpha}{4\pi}\right)^2 \left[+\frac{9}{32}\ln^4\left(\frac{Q^2}{M^2}\right) - \frac{19}{48}\ln^3\left(\frac{Q^2}{M^2}\right) - \left(-\frac{7}{8}\pi^2 + \frac{463}{48}\right)\ln^2\left(\frac{Q^2}{M^2}\right) + \left(\frac{39}{2}\frac{\mathrm{Cl}_2\left(\frac{\pi}{3}\right)}{\sqrt{3}} + \frac{45}{4}\frac{\pi}{\sqrt{3}} - \frac{61}{2}\zeta_3 - \frac{11}{24}\pi^2 + 29\right)\ln\left(\frac{Q^2}{M^2}\right) \right]$$



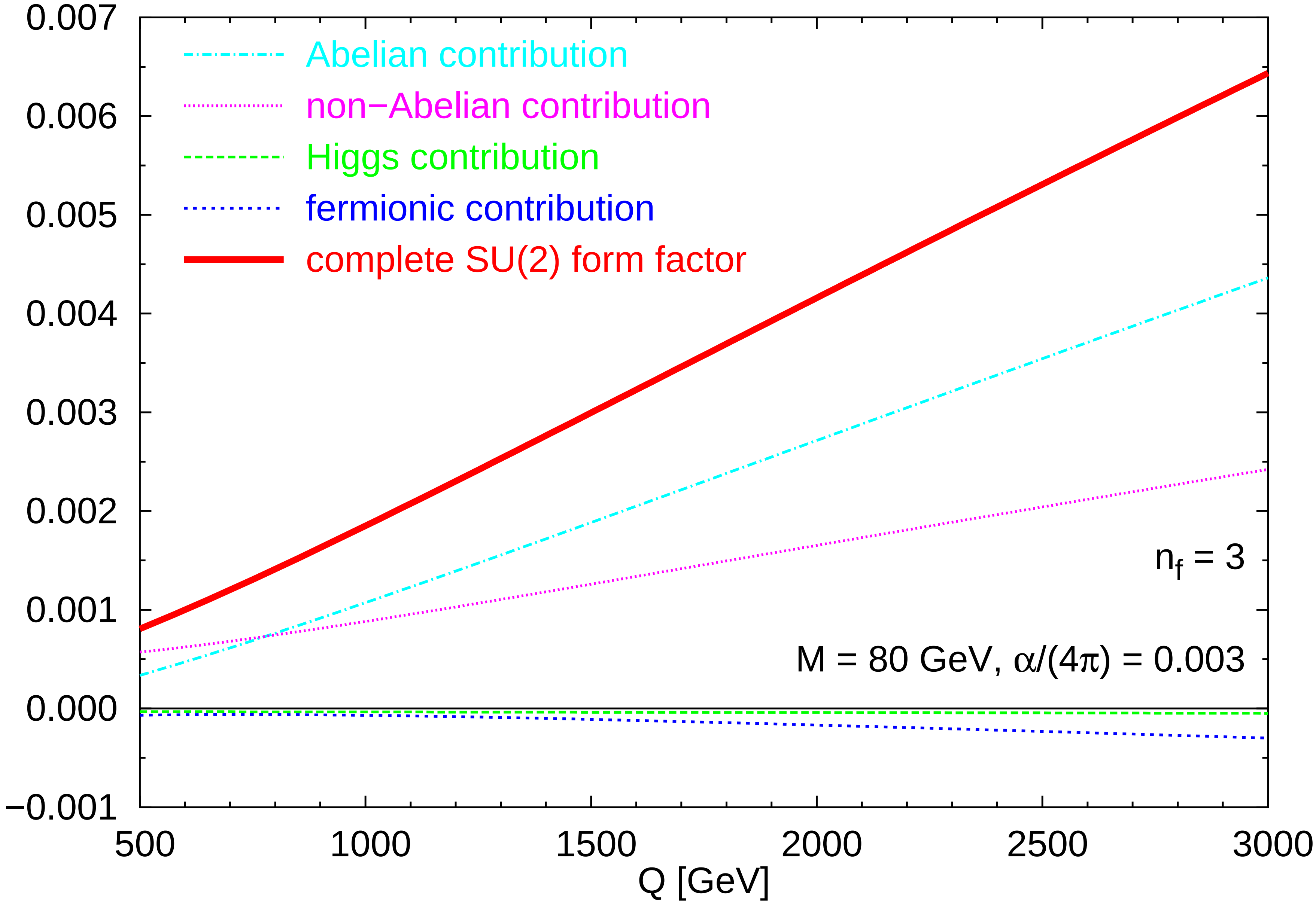
N^3LL approximation

$M_{\text{Higgs}} = M$

$n_f = 3$

Massive SU(2) form factor in 2-loop approximation: individual contributions

(N^{3}LL approximation, $M_{\text{Higgs}} = M$, $n_f = 3$, Feynman-'t Hooft gauge)



IV Summary & outlook

Massive SU(2) form factor

- weak interaction with massive gauge bosons
- 2-loop result in N^3LL approximation ✓

⇒ precise control of radiative corrections

to be mentioned also

- factorization of IR singularities in massive SU(2) $\times$ massless U(1) ✓
- expansion in the mass difference $M_W \approx M_Z$ possible ✓

Outlook

- 4-fermion scattering amplitude $f\bar{f} \to f'\bar{f}'$
- electroweak corrections to the cross section, ...