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Sudakov logarithms in a massive U(1) theory in two-loop approximation

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- Motivation
- Sudakov logarithms
- U(1) form factor in two-loop approximation
- Summary

Motivation

Electroweak precision physics

- experimentally measured by now at energy scales up to $\sim M_{W,Z}$
- future generation of accelerators $\to$ TeV region

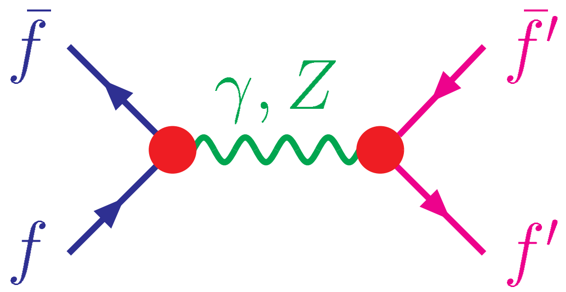
Electroweak radiative corrections

Kühn et al. '00; Fadin et al. '00;
Denner, Pozzorini '01

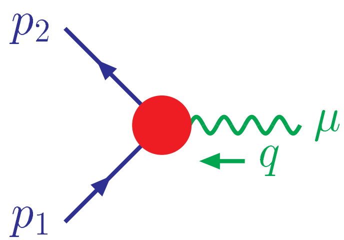
at high momentum transfer $Q \sim \text{TeV} \gg M_{W,Z}$

large negative corrections in *exclusive* cross sections

Important class of processes: four-fermion scattering



Form factor F describes vertex corrections:



$$= \bar{u}(p_2)\,\gamma^\mu\,u(p_1)\cdot F + \ldots$$

Perturbation theory

Expand the form factor in $\alpha \ll 1$:

$$F = F_0 + \alpha\,F_1 + \alpha^2\,F_2 + \ldots$$

Why do we need higher order corrections?

- $F_0 = 1 \to$ Born term, leading order contribution (LO)
- $F_1 \to$ next-to-leading order (NLO), one-loop correction
  $\hookrightarrow$ renormalization introduces *scale dependence*: $\alpha(\mu)\,F_1$
- $F_2 \to$ next-to-next-leading order (NNLO), two-loop correction
  $\hookrightarrow$ reduces scale variation $\to$ essential for precision calculations

Massive U(1) model

Standard Model
$SU(2)_L \otimes U(1)$
$M_W \neq M_Z,\ M_\gamma = 0$

$\longrightarrow$

SU(2) model
non-Abelian
$M_W = M_Z$

$\longrightarrow$

U(1) model
Abelian ($\sim$ QED)
gauge boson mass M

$\nwarrow \quad \uparrow \quad \nearrow$

spontaneous symmetry breaking
Higgs contributions

High energy behaviour $\rightarrow$ *Sudakov limit*

- momentum transfer $\left|q^2\right| \equiv Q^2 \gg M^2$
  $\hookrightarrow$ neglect terms suppressed by a factor of M^2/Q^2
  $\rightarrow$ *logarithmic approximation*
- neglect fermion masses $\rightarrow$ external on-shell fermions: $p_1^2 = p_2^2 = 0$

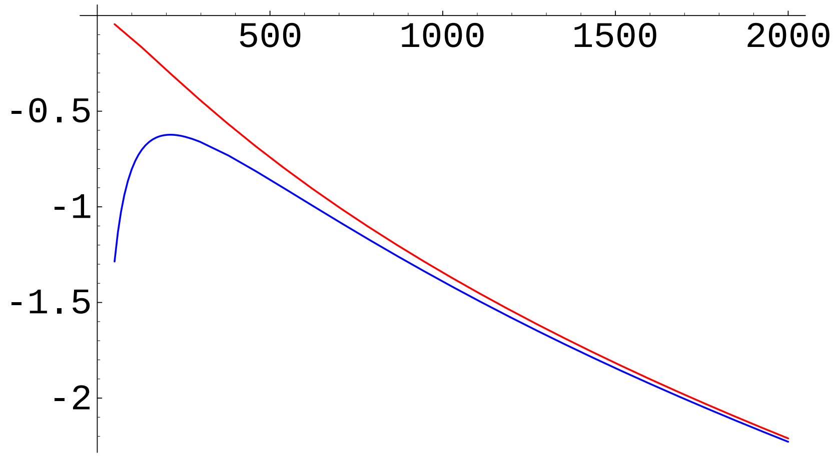
Sudakov logarithms

One-loop correction to the U(1) form factor

$$\alpha\, F_1 = \frac{\alpha}{4\pi}\left\{\left(1-\frac{M^2}{Q^2}\right)^2\left[-\ln^2\left(\frac{Q^2}{M^2}\right) - 2\ln\left(\frac{Q^2}{M^2}\right)\ln\left(1-\frac{M^2}{Q^2}\right) + 2\,\mathrm{Li}_2\left(\frac{M^2}{Q^2}\right) - \frac{2\pi^2}{3}\right] + \left(3-2\frac{M^2}{Q^2}\right)\ln\left(\frac{Q^2}{M^2}\right) - \frac{7}{2} + 2\frac{M^2}{Q^2}\right\}$$

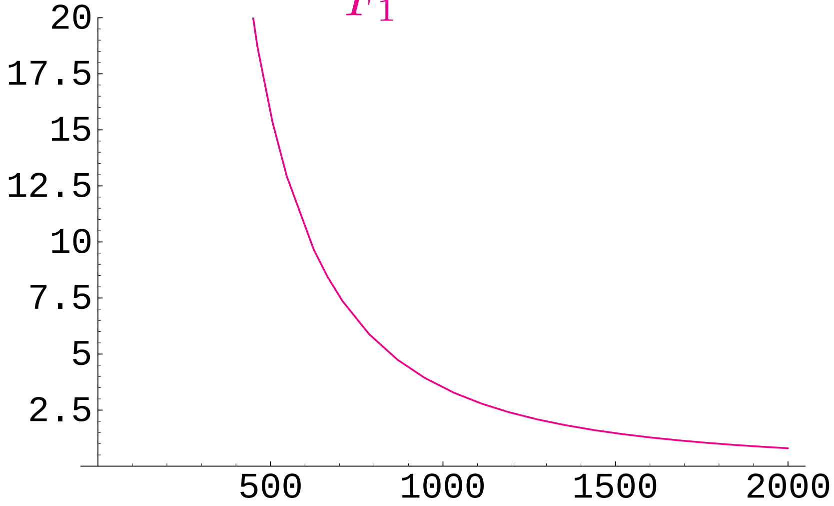
Logarithmic approximation: $\alpha\, F_1\big|_{\text{logs}} = \frac{\alpha}{4\pi}\left[-\ln^2\left(\frac{Q^2}{M^2}\right) + 3\ln\left(\frac{Q^2}{M^2}\right) - \frac{2\pi^2}{3} - \frac{7}{2}\right]$

F_1 (exact), $F_1\big|_{\text{logs}}$



Q [GeV]

$\frac{F_1 - F_1|_{\text{logs}}}{F_1}$ [%]

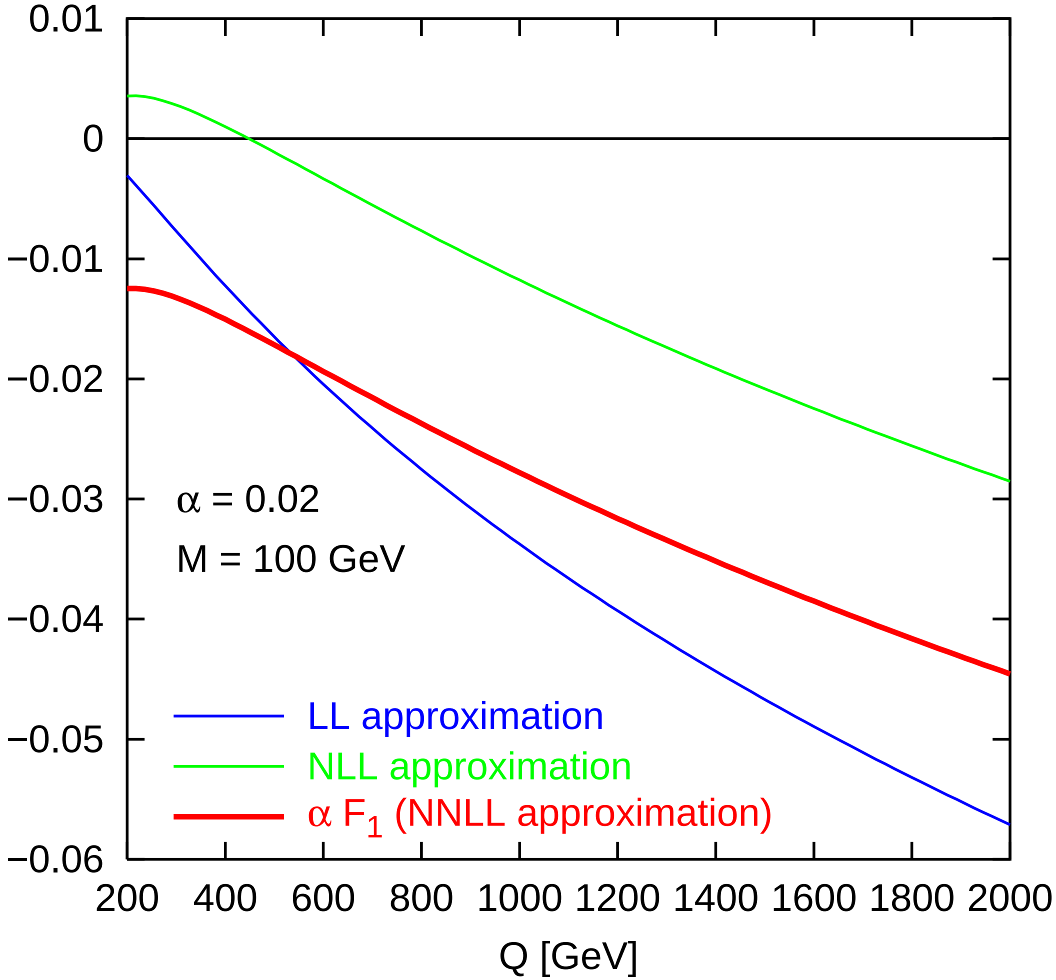


Q [GeV]

$M = 100$ GeV

Logarithmic approximation of the U(1) form factor

$$\alpha F_1\Big|_{\text{logs}} = \frac{\alpha}{4\pi}\Bigg[\overbrace{\underbrace{-\ln^2\left(\frac{Q^2}{M^2}\right)}_{\text{LL}} + 3\ln\left(\frac{Q^2}{M^2}\right)}^{\text{NLL}} - \frac{2\pi^2}{3} - \frac{7}{2}\Bigg]$$



Double logarithmic corrections

Sudakov '56 (QED)

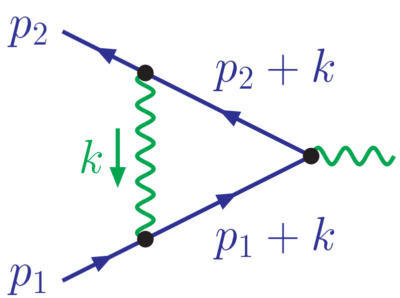
→ Sudakov logarithms

↪ *large* for $Q^2 \gg M^2$

$Q \sim 1$ TeV → $\ln^2 \sim \ln^1 \sim \ln^0$

↪ need *all* terms of the logarithmic approximation

Where do double logarithms arise from?



$p_1^2 = p_2^2 = 0$

$(p_{1,2}^\mu) = (E_f, \pm\vec{p})$

$(k^\mu) = (E_b, \vec{k}\,)$

Simplified loop integration, $M = 0$:

$$\int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2 + i\varepsilon} \frac{1}{(p_1 + k)^2 + i\varepsilon} \frac{1}{(p_2 + k)^2 + i\varepsilon}$$

Gauge boson propagator: $\dfrac{1}{k^2 + i\varepsilon} \sim -i\pi\,\delta(k^2) \Rightarrow$ contribution from $k^2 = 0$

$\hookrightarrow$ Fermion propagators: $\dfrac{1}{(p_{1,2} + k)^2} \sim \dfrac{1}{2E_f E_b (1 \mp \cos\theta_{pk})}$

$\rightarrow$ Loop integration is singular at $E_b = 0$ and $\cos\theta_{pk} = \pm 1$ (i.e. $\vec{k} \| \vec{p}_{1,2}$)

$$\int \frac{d^4k}{(2\pi)^4} \longrightarrow \int \frac{dE_b}{E_b} \int \frac{d(\cos\theta_{pk})}{1 \mp \cos\theta_{pk}} \sim \ln(\ldots)\,\ln(\ldots)$$

Resummation of the logarithms

Large radiative corrections in the TeV region
$\rightarrow$ Systematic treatment of higher order corrections needed.

Evolution equation for the form factor   Sen '81; Collins '89; Korchemsky '89; ...
in logarithmic approximation $\rightsquigarrow$ sum up logarithms to all orders in α:

$$\frac{\partial F(Q^2)}{\partial \ln Q^2} = \left[\int_{M^2}^{Q^2} \frac{dx}{x} \gamma\big(\alpha(x)\big) + \zeta\big(\alpha(Q^2)\big) + \xi\big(\alpha(M^2)\big) \right] F(Q^2)$$

Solution:

$$F(Q^2) = \tilde{F}\big(\alpha(M^2)\big) \exp\left\{ \int_{M^2}^{Q^2} \frac{dx}{x} \left[\int_{M^2}^{x} \frac{dx'}{x'} \gamma\big(\alpha(x')\big) + \zeta\big(\alpha(x)\big) + \xi\big(\alpha(M^2)\big) \right] \right\}$$

Schematically:

$$F = (1 + \alpha \cdot \text{const} + \alpha^2 \cdot \text{const} + \ldots) \exp\Big(\alpha\,(\ln^2 + \ln) + \alpha^2\,(\ln^3 + \ln^2 + \ln) + \ldots \Big)$$

Coefficients involve anomalous dimensions γ, ζ, ξ
and running of $\alpha(\mu)$ ($\rightarrow \beta$ function).

Resummation (2)

$$F = 1 + \alpha\,(\ln^2 + \ln + \text{const}) + \alpha^2\,(\ln^4 + \ln^3 + \ln^2 + \ln + \text{const}) + \ldots$$

$$= (1 + \alpha \cdot \text{const} + \alpha^2 \cdot \text{const} + \ldots)\,\exp\Big(\alpha\,(\ln^2 + \ln) + \alpha^2\,(\ln^3 + \ln^2 + \ln) + \ldots\Big)$$

Improved perturbation series:

- $\exp(\ldots)$ contains all large logarithms
- prefactor $\sim$ const

Leading logarithm in $\mathcal{O}(\alpha^n)$: $\left[\alpha \ln^2\left(\frac{Q^2}{M^2}\right)\right]^n$

Match expansion of $\exp(\ldots)$ with known results from perturbation theory
$\rightarrow$ extract coefficients of anomalous dimensions
$\hookrightarrow$ obtain leading (and subleading) logarithms of higher order corrections.

Knowledge of anomalous dimensions for massive SU(N) and U(1) models:

- ζ and ξ up to $\mathcal{O}(\alpha)$
- γ up to $\mathcal{O}(\alpha^2)$ Kodaira, Trentadue '81

$\Rightarrow$ NNLL approximation of F_2 known.

U(1) form factor in two-loop approximation

Known from resumption:

(without Higgs contributions)

Kühn et al. '01

$$\alpha^2 F_2 = \frac{\alpha^2}{(4\pi)^2}\left[+\frac{1}{2}\ln^4\left(\frac{Q^2}{M^2}\right) - \left(\frac{4}{9}n_f + 3\right)\ln^3\left(\frac{Q^2}{M^2}\right) + \left(\frac{38}{9}n_f + \frac{2\pi^2}{3} + 8\right)\ln^2\left(\frac{Q^2}{M^2}\right) + \ldots\right]$$

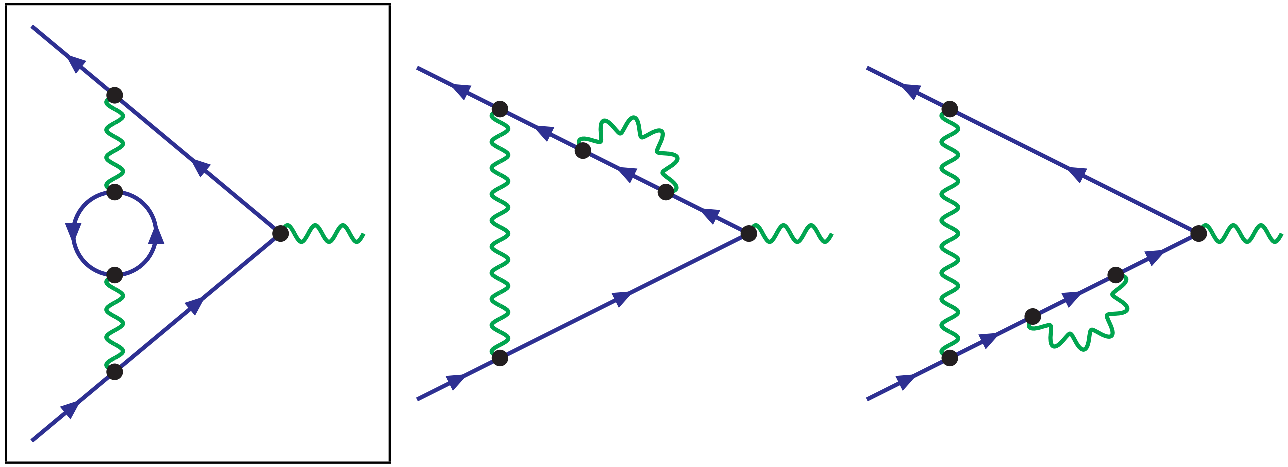
($n_f = \#$ fermions)

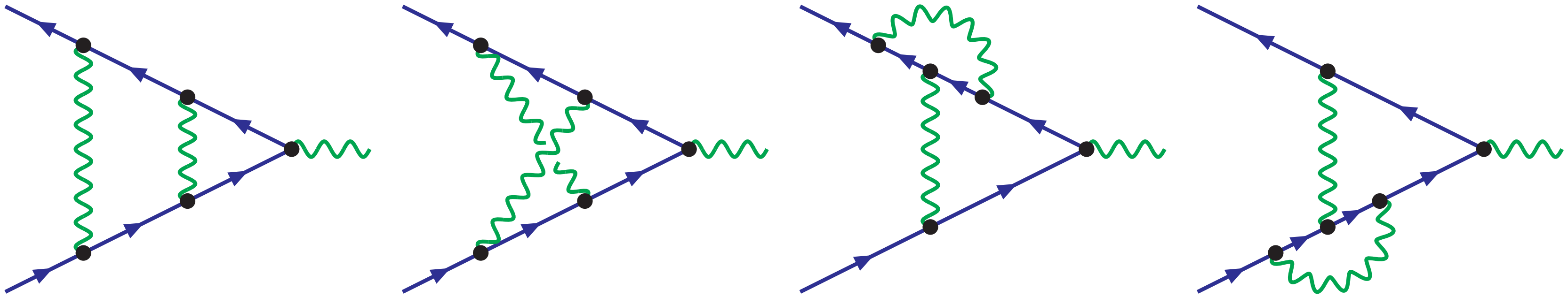
- NNLL approximation known, but $\ln^1$ and $\ln^0$ missing
- $Q \sim 1\,\text{TeV} \to \ln^4 \sim \ln^3 \sim \ln^2$
- alternating sign $\to$ cancellation between logarithmic terms
- contribution from missing terms?

Complete two-loop corrections in logarithmic approximation necessary.

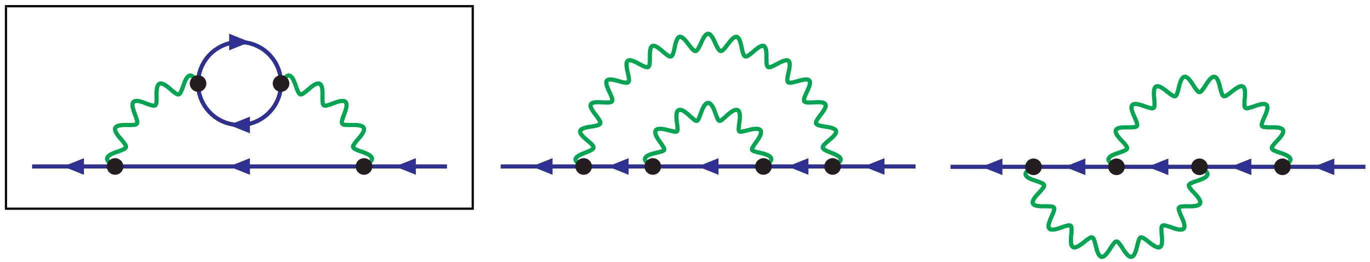
Two-loop contributions to the U(1) form factor (without Higgs contributions)

Vertex corrections:





Self energy corrections:



□ = contributions $\propto n_f$

Calculation of the feynman diagrams – some aspects

Tensor reduction of feynman integrals

Aim: tensor integrals $\rightarrow$ scalar integrals

<u>One-loop integrals:</u> Passarino, Veltman '79

$$\int \frac{d^D k}{(2\pi)^D} \frac{k^\mu\, k^\nu}{k^2 (k+p)^2} = g^{\mu\nu} B_{00}(p^2) + p^\mu p^\nu B_{11}(p^2)$$

- determine set of Lorentz tensors ($g^{\mu\nu}$ and external momenta)
- extract the coefficients: contractions $\rightarrow$ set of linear equations

$\hookrightarrow$ not possible for all two-loop topologies

<u>Two-loop integrals:</u>

$\rightarrow$ need alternatives

Two-loop tensor reduction

Tarasov '97; Anastasiou et al. '00

- introduce Schwinger parameters:

$$\frac{1}{(k_i^2)^{\nu_i}} = \frac{(-1)^{\nu_i}}{\Gamma(\nu_i)} \int_0^\infty dx_i \, x_i^{\nu_i - 1} \, e^{x_i k_i^2}$$

- relatively simple tensor reduction, e.g.:

$$\int \frac{d^D k}{i\pi^{D/2}} \, k^\mu k^\nu \, e^{ak^2} = -\frac{1}{2a} \, g^{\mu\nu} \int \frac{d^D k}{i\pi^{D/2}} \, e^{ak^2}$$

- map factors within parameter integrals to operators which
  - increase powers of propagators: $\mathbf{i}^+ \, \frac{1}{(k_i^2)^{\nu_i}} = \frac{1}{(k_i^2)^{\nu_i+1}}$
  - increase the space-time dimension: $\mathbf{d}^+ \int \frac{d^D K}{i\pi^{D/2}} = \int \frac{d^{D+2} K}{i\pi^{(D+2)/2}}$

Calculation of scalar integrals - some methods

- Partial integration: reduce powers of propagators

$$\int d^D k \, \frac{\partial}{\partial k^\mu} \, \frac{k^\mu}{\text{(propagators)}} = 0$$

  and similar equations $\to$ relations between integrals.

  Repeated application $\to$ simpler topologies.

- Feynman parameters: combine propagators

$$\frac{1}{(k^2)^{\nu_1} \, (\ell^2)^{\nu_2}} = \frac{\Gamma(\nu_1 + \nu_2)}{\Gamma(\nu_1) \, \Gamma(\nu_2)} \int\limits_0^1 dx \, \frac{x^{\nu_1 - 1} \, (1-x)^{\nu_2 - 1}}{[x \, k^2 + (1-x)\ell^2]^{\nu_1 + \nu_2}}$$

- Mellin-Barnes representation: massive $\to$ massless propagators

$$\frac{1}{(k^2 - M^2)^\nu} = \frac{1}{\Gamma(\nu)} \int\limits_{-i\infty}^{i\infty} \frac{ds}{2\pi i} \, \frac{(-M^2)^s}{(k^2)^{\nu+s}} \, \Gamma(-s) \, \Gamma(\nu + s)$$

U(1) form factor in two-loop approximation

Consider only contributions $\propto n_f = \#$ fermions B.F., Kühn, Moch

$\hookrightarrow$ n_f-part is separately gauge invariant and renormalizable.

- renormalization of the coupling α in the $\overline{\text{MS}}$ scheme at the scale $\mu^2 = M^2$
- renormalization of the gauge boson mass M in the on-shell scheme

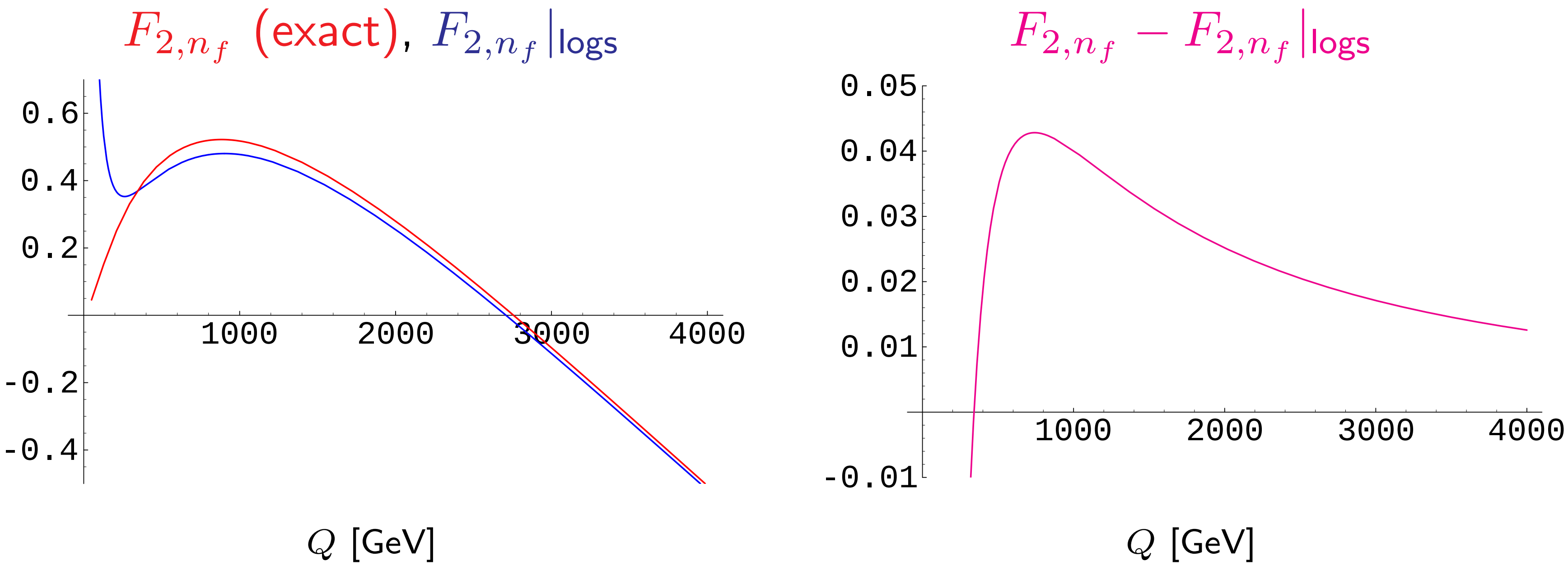
Result:

$$\alpha^2 F_{2,n_f} = \frac{\alpha^2 n_f}{(4\pi)^2}\left\{\left(1-\frac{M^2}{Q^2}\right)\left(1-3\frac{M^2}{Q^2}\right)\left[-\frac{4}{9}\ln^3\left(\frac{Q^2}{M^2}\right)-\frac{4}{3}\ln^2\left(\frac{Q^2}{M^2}\right)\ln\left(1-\frac{M^2}{Q^2}\right)\right.\right.$$
$$\left.+\frac{8}{3}\ln\left(\frac{Q^2}{M^2}\right)\mathrm{Li}_2\left(\frac{M^2}{Q^2}\right)+\frac{8}{3}\mathrm{Li}_3\left(\frac{M^2}{Q^2}\right)\right]$$
$$+\left(1-\frac{M^2}{Q^2}\right)^2\left[\frac{16}{9}\ln\left(\frac{Q^2}{M^2}\right)\ln\left(1-\frac{M^2}{Q^2}\right)-\frac{16}{9}\mathrm{Li}_2\left(\frac{M^2}{Q^2}\right)+\frac{16\pi^2}{27}\right]$$
$$\left.+\left(\frac{38}{9}-\frac{52}{9}\frac{M^2}{Q^2}+\frac{8}{9}\frac{M^4}{Q^4}\right)\ln^2\left(\frac{Q^2}{M^2}\right)-\left(\frac{34}{3}-\frac{88}{9}\frac{M^2}{Q^2}\right)\ln\left(\frac{Q^2}{M^2}\right)+\frac{115}{9}-\frac{88}{9}\frac{M^2}{Q^2}\right\}$$

Logarithmic approximation:

$$\alpha^2 F_{2,n_f}\Big|_{\text{logs}} = \frac{\alpha^2 n_f}{(4\pi)^2}\left[-\frac{4}{9}\ln^3\left(\frac{Q^2}{M^2}\right)+\frac{38}{9}\ln^2\left(\frac{Q^2}{M^2}\right)-\frac{34}{3}\ln\left(\frac{Q^2}{M^2}\right)+\frac{16\pi^2}{27}+\frac{115}{9}\right]$$

Exact n_f form factor $\leftrightarrow$ logarithmic approximation

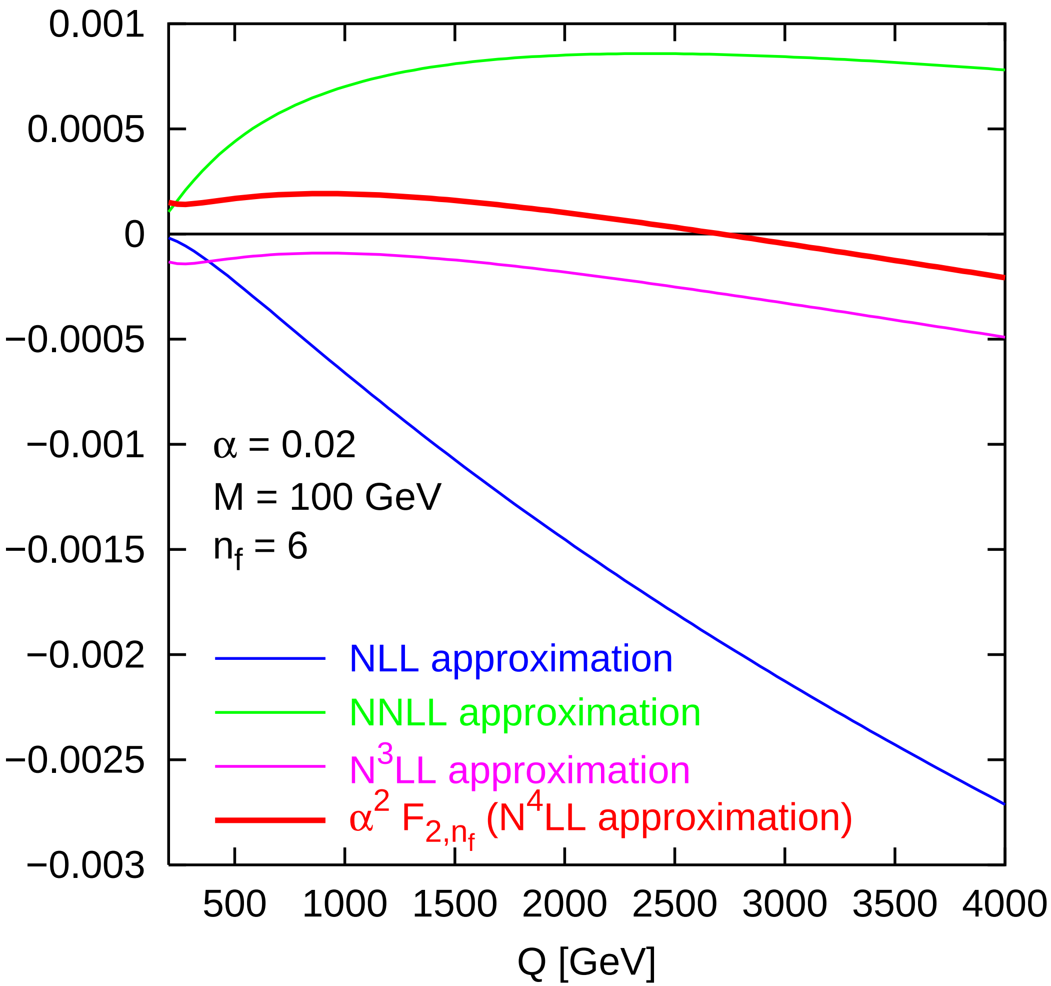


$M = 100\,\text{GeV}$, $n_f = 6$

Maximum error of logarithmic approximation $< 10\%$ (for $Q \geq 400\,\text{GeV}$)

Logarithmic approximation of F_{2,n_f}

$$\alpha^2 F_{2,n_f}\Big|_{\text{logs}} = \frac{\alpha^2 n_f}{(4\pi)^2}\left[\underbrace{\overbrace{-\frac{4}{9}\ln^3\left(\frac{Q^2}{M^2}\right)}_{\text{NLL}}+\frac{38}{9}\ln^2\left(\frac{Q^2}{M^2}\right)}^{\text{NNLL}}-\frac{34}{3}\ln\left(\frac{Q^2}{M^2}\right)}_{\text{N}^3\text{LL}}+\frac{16\pi^2}{27}+\frac{115}{9}\right]$$



No $\ln^4$ in n_f-part:

LL determined by F_1 via

$F \sim \exp\left[-\frac{\alpha}{4\pi}\ln^2\left(\frac{Q^2}{M^2}\right)\right]$

Alternating sign continues

$\rightarrow$ large cancellation

$\hookrightarrow$ asymptotic behaviour comes in very late!

F_{2,n_f} is only determined by the *complete* logarithmic approximation (even qualitatively)!

Summary

- The two-loop contributions of the n_f-part in the U(1) form factor have been calculated *exactly* and in complete *logarithmic approximation*.
- For energies in the TeV region, the form factor is quite well described by the logarithmic approximation.
- Large cancellations between the logarithmic terms.
- All terms of the logarithmic approximation are needed, up to $\ln^0$.

Outlook

- Full two-loop contributions to the U(1) form factor.
  Exact calculation impossible $\rightarrow$ logarithmic approximation
  $\rightarrow$ work in progress ...
- Higgs-contributions: similar to n_f part.
  Dependence on the Higgs mass? $\rightarrow$ work in progress ...
- Extension to other gauge theories: SU(2), Standard Model.