PAUL SCHERRER INSTITUT

SPS Annual Meeting in Genève, March 26–27, 2008

Two-loop electroweak NLL corrections with massless and massive fermions

Bernd Jantzen

Paul Scherrer Institut (PSI), Villigen

In collaboration with **Ansgar Denner** and **Stefano Pozzorini**

I Electroweak corrections at high energies
II Two-loop next-to-leading logarithmic (NLL) corrections
III Results for massless and massive fermionic processes
IV Summary & outlook

I Electroweak corrections at high energies

Electroweak (EW) collider physics

- up to now (LEP, Tevatron) at energy scales $\lesssim M_{\mathrm{W,Z}}$
- future colliders (LHC, ILC/CLIC) $\longrightarrow$ reach TeV regime

  $\hookrightarrow$ new energy domain $\sqrt{s} \gg M_{\mathrm{W}}$ becomes accessible!

EW radiative corrections at high energies $\sqrt{s} \gg M_{\mathrm{W}}$

$\Rightarrow$ enhanced by large Sudakov logarithms

$$\text{per loop:} \quad \ln^2\left(\frac{s}{M_{\mathrm{W}}^2}\right) \sim 25 \quad \text{at } \sqrt{s} \sim 1\,\mathrm{TeV}$$

- $M_{W,Z} \neq 0 \longrightarrow$ exclusive observables possible: only virtual W's and Z's

  ($\neq$ QED: singular logs cancel between virtual and real corrections)
- large logs even in inclusive observables (Bloch–Nordsieck violations)

General form of EW corrections for $s \gg M_\mathrm{W}^2$

$$\left[L = \ln\left(\frac{s}{M_\mathrm{W}^2}\right)\right]$$

$\hookrightarrow$ logarithmic approximation, Sudakov approximation:

1 loop:

$$\alpha\left[C_1^{\mathrm{LL}}\,L^2 + C_1^{\mathrm{NLL}}\,L + C_1^{\mathrm{N^2LL}}\right] + \mathcal{O}\left(\frac{M_\mathrm{W}^2}{s}\right)$$

$$\downarrow \; -17\,\% \qquad \downarrow \; +12\,\% \qquad \downarrow \; -3\,\%$$

2 loops:

$$\alpha^2\left[C_2^{\mathrm{LL}}\,L^4 + C_2^{\mathrm{NLL}}\,L^3 + C_2^{\mathrm{N^2LL}}\,L^2 + C_2^{\mathrm{N^3LL}}\,L + C_2^{\mathrm{N^4LL}}\right] + \mathcal{O}\left(\frac{M_\mathrm{W}^2}{s}\right)$$

$$\downarrow \; +1.7\,\% \qquad \downarrow \; -1.8\,\% \qquad \downarrow \; +1.2\,\% \qquad \downarrow \; -0.3\,\%$$

[$\sigma(u\bar{u} \to d\bar{d})$ @ $\sqrt{s} = 1$ TeV, B.J., Kühn, Penin, Smirnov '05]

For theoretical predictions with accuracy $\sim 1\,\%$:

$\Rightarrow$ 2-loop corrections important

$\Rightarrow$ LL approximation not sufficient

With massless photons: log $\rightsquigarrow$ $1/\epsilon$ in $D = 4 - 2\epsilon$ dimensions

Virtual 2-loop EW corrections

Resummation of 1-loop results:

- LL & NLL for arbitrary processes ($M_Z = M_W$) — Fadin, Lipatov, Martin, Melles '99; Melles '00, '01
- N^{2}LL for $f\bar{f} \to f'\bar{f}'$ ($m_f = 0$, $M_Z = M_W$) — Kühn, Penin, Smirnov '99, '00; Kühn, Moch, Penin, Smirnov '01
- N^{2}LL for $e^+e^- \to \mathrm{W}^+\mathrm{W}^-$ — Kühn, Metzler, Penin '07
- SCET method — Chiu, Golf, Kelley, Manohar '07

$\longrightarrow$ apply evolution equations to spontaneously broken Standard Model

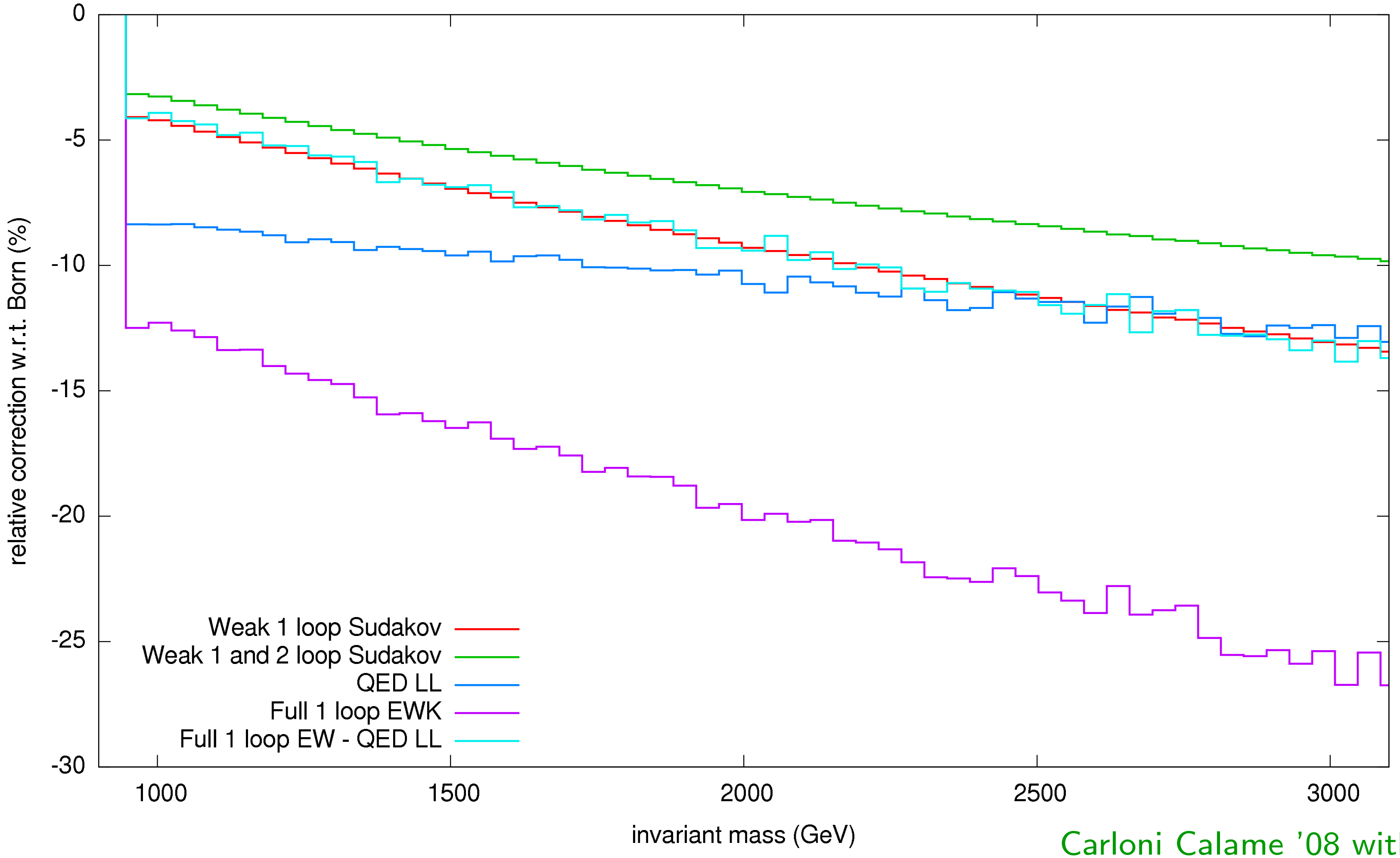
$\hookrightarrow$ Assumption: splitting into symmetric SU(2)×U(1) regime & QED regime possible

Diagrammatic 2-loop calculations $\to$ check & extend resummation results:

- LL & angular-dependent NLLs for arbitrary processes — Melles '00; Hori, Kawamura, Kodaira '00; Beenakker, Werthenbach '00, '01; Denner, Melles, Pozzorini '03
- NLL for fermionic processes ($m_f = 0$, $M_Z \neq M_W$) — Pozzorini '04; Denner, B.J., Pozzorini '06
- N^{3}LL for fermionic form factor ($m_f = 0$, $M_Z = M_W$)
  $\hookrightarrow$ N^{3}LL for $f\bar{f} \to f'\bar{f}'$ ($m_f = 0$, $M_Z \approx M_W$) via evolution equations — B.J., Kühn, Moch '03; B.J., Kühn, Penin, Smirnov '04, '05

EW corrections at the LHC

Drell–Yan $pp \to \mu^+\mu^- + X$: (electro)weak 1-loop & 2-loop corrections



Carloni Calame '08 with HORACE
and Sudakov results from
B.J., Kühn, Penin, Smirnov '05

$\Rightarrow$ Sudakov approximation very good at high energies

$\Rightarrow$ 2-loop effects $\sim \mathcal{O}(\%)$

II Two-loop next-to-leading logarithmic corrections

Goal: virtual 2-loop EW corrections for arbitrary processes in NLL accuracy
$\hookrightarrow$ **start with** processes involving massless & massive external fermions

Parameters:

- different large kinematical invariants $r_{ij} = (p_i + p_j)^2 \sim Q^2 \gg M_\mathrm{W}^2$
- different heavy particle masses $M_\mathrm{W}^2 \sim M_\mathrm{Z}^2 \sim m_\mathrm{t}^2 \sim M_\mathrm{Higgs}^2$
- massive (top quark) and massless fermions

$\Rightarrow$ logs $L = \ln\left(\frac{Q^2}{M_\mathrm{W}^2}\right)$ and $\frac{1}{\epsilon}$ poles (from virtual photons)

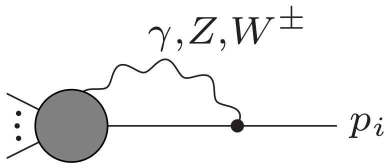
1 loop: LL $\to \epsilon^{-2},\ L\epsilon^{-1},\ L^2,\ L^3\epsilon,\ L^4\epsilon^2$; NLL $\to \epsilon^{-1},\ L,\ L^2\epsilon,\ L^3\epsilon^2$

2 loops: LL $\to \epsilon^{-4},\ L\epsilon^{-3},\ L^2\epsilon^{-2},\ L^3\epsilon^{-1},\ L^4$; NLL $\to \epsilon^{-3},\ L\epsilon^{-2},\ L^2\epsilon^{-1},\ L^3$

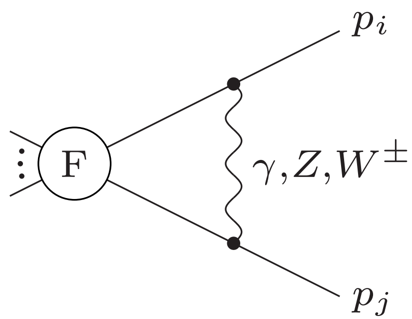
$\Rightarrow$ NLL coefficients involve small logs $\ln\left(\frac{-r_{ij}}{Q^2}\right)$ and $\ln\left(\frac{M_\mathrm{Z}^2, m_\mathrm{t}^2}{M_\mathrm{W}^2}\right)$

Extraction of NLL logs at 1 loop

Logs originate from mass singularities
in collinear/soft regions:
(+ UV logs)

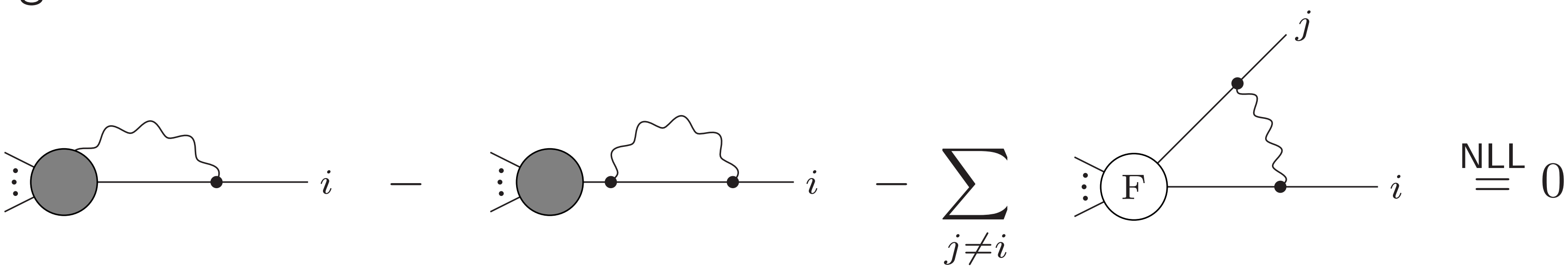


Isolate factorizable contributions:



$\hookrightarrow$ separate loop integral from Born diagram  via soft–collinear approximation

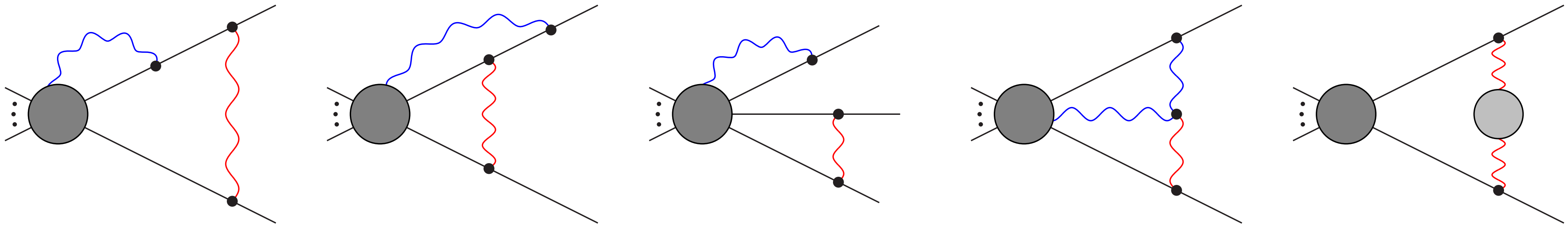
Remaining non-factorizable contributions: collinear Ward identities Denner, Pozzorini '01



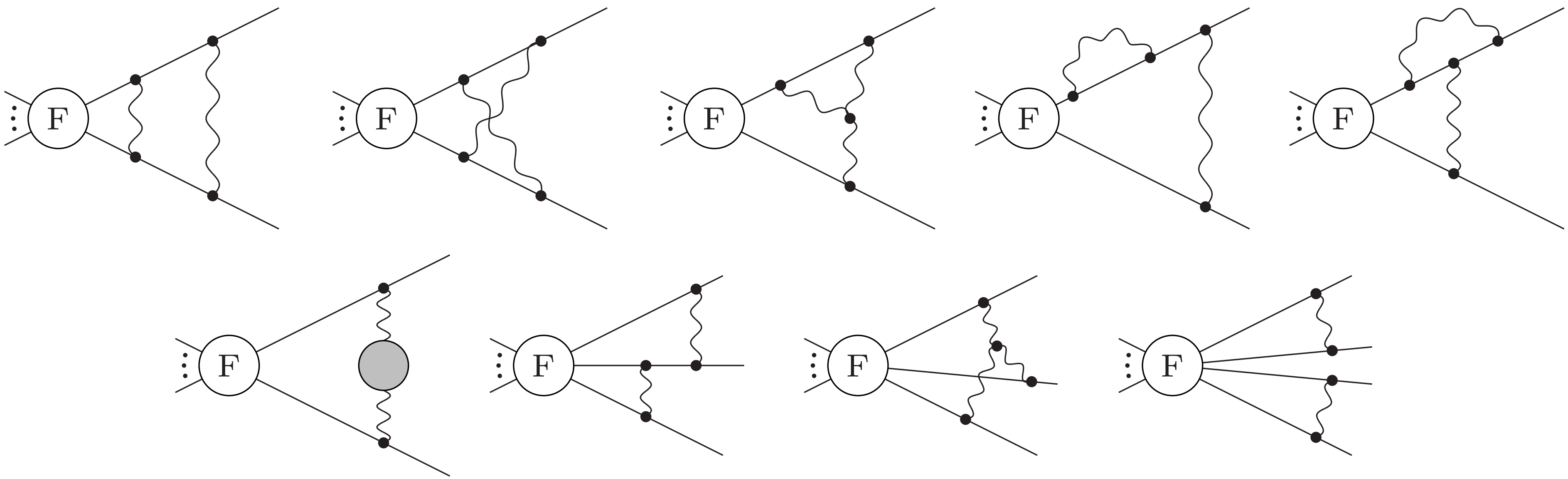
Factorizable contributions contain <u>all</u> soft & collinear NLL mass singularities.

Extraction of NLL logs at 2 loops

↪ contributions: soft×soft and soft×collinear (without Yukawa contributions):



Factorizable contributions:



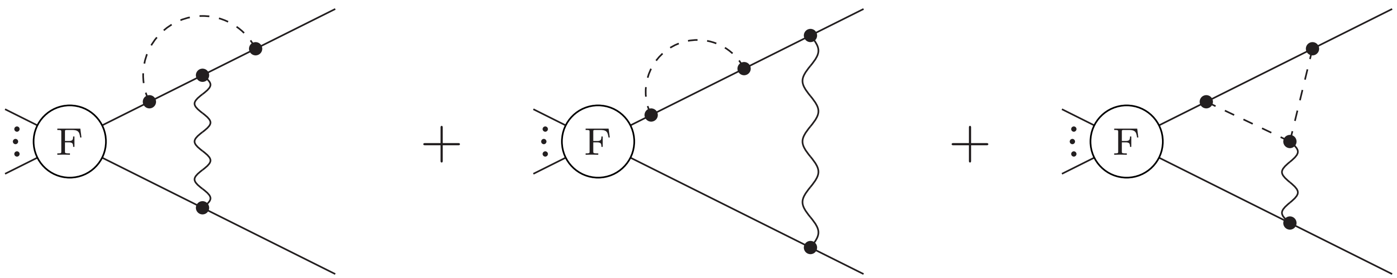
- calculated with soft–collinear approximation (and projection techniques)
- non-factorizable contributions vanish

Denner, B.J., Pozzorini '06

Yukawa contributions

Massive fermions $\longrightarrow$ Yukawa couplings to scalars (Higgs, Goldstone bosons)

- many Yukawa contributions are suppressed (soft/collinear limit, $M_W^2/Q^2 \to 0$)
- only <u>three</u> non-suppressed diagrams with several external legs:



$$+ \quad + \quad \stackrel{\text{NLL}}{=} 0$$

Sum vanishes due to gauge invariance of Yukawa coupling

$\hookrightarrow$ NLL Yukawa contributions only in field-strength renormalization

III Results for massless and massive fermionic processes

Factorizable contributions

loop integrals calculated with two independent methods:

- automatized algorithm based on sector decomposition — Denner, Pozzorini '04
- combination of expansion by regions & Mellin–Barnes representations — B.J., Smirnov '06 & refs. therein

Result for amplitude of fermionic processes $f_1 f_2 \to f_3 \cdots f_n$ in $\mathcal{O}(\alpha^2)$

$$\mathcal{M} \overset{\text{NLL}}{=} \underbrace{\exp\left(\frac{1}{2}\sum_{\substack{i=1\\ j\neq i}}^{n} \delta_{ij}^{\text{em}}\right)}_{\text{elektromagnetic}} \underbrace{\exp\left(\frac{1}{2}\sum_{\substack{i=1\\ j\neq i}}^{n} \delta_{ij}^{\text{sew}}\right)}_{\substack{\text{symmetric-electroweak}\\ M_\gamma = M_\text{Z} = M_\text{W}}} \underbrace{\left(1+\frac{1}{2}\sum_{\substack{i=1\\ j\neq i}}^{n} \delta_{ij}^{\text{Z}}\right)}_{\text{from } M_\text{Z}\neq M_\text{W}} \underbrace{\mathcal{M}_0}_{\text{Born}}$$

- universal result: δ_{ij}^{sew}, δ_{ij}^{em}, δ_{ij}^{Z} depend only on external quantum numbers
- electromagnetic singularities (in δ_{ij}^{em}) factorized $\to$ separable

Symmetric-electroweak terms: independent of fermion masses

$$\delta_{ij}^{\text{sew}} = -\frac{\alpha}{4\pi}\sum_{V=\mathrm{B},\mathrm{W}^a} I_i^{\bar{V}} I_j^V\, I_{ij}(\epsilon, M_\mathrm{W}) - \frac{\alpha^2}{(4\pi)^2}\left(\frac{g_1^2}{e^2}\frac{Y_iY_j}{4}b_1^{(1)} + \frac{g_2^2}{e^2}T_i^aT_j^a b_2^{(1)}\right) J_{ij}(\epsilon, M_\mathrm{W}, \mu^2)\,,$$

$$I_{ij}(\epsilon, M_\mathrm{W}) = -L^2 - \frac{2}{3}L^3\epsilon - \frac{1}{4}L^4\epsilon^2 + \left[\frac{3}{2} - \ln\left(\frac{-r_{ij}}{Q^2}\right) - \underbrace{\frac{y_i^{\kappa_i}}{C_i^{\text{ew}}}\frac{g_2^2 m_\mathrm{t}^2}{8e^2M_\mathrm{W}^2}}_{\text{Yukawa term}}\right]\left(2L + L^2\epsilon + \frac{1}{3}L^3\epsilon^2\right),$$

$$J_{ij}(\epsilon, M, \mu^2) = \frac{1}{\epsilon}\left[I_{ij}(2\epsilon, M) - \left(\frac{Q^2}{\mu^2}\right)^\epsilon I_{ij}(\epsilon, M)\right]$$

Electromagnetic terms: QED$(M_\gamma = 0)$ − QED$(M_\gamma = M_\mathrm{W})$ $[\mu^2 = M_\mathrm{W}^2]$

$$\delta_{ij}^{\text{em}} = -Q_iQ_j\left\{\frac{\alpha}{4\pi}\left[I_{ij}(\epsilon,0) - I_{ij}(\epsilon, M_\mathrm{W})\right] + \frac{\alpha^2}{(4\pi)^2}b_{\text{QED}}^{(1)}\left[J_{ij}(\epsilon, 0, M_\mathrm{W}^2) - J_{ij}(\epsilon, M_\mathrm{W}, M_\mathrm{W}^2)\right]\right\},$$

$$I_{ij}(\epsilon, 0) = -\left(2\epsilon^{-2} + 3\epsilon^{-1}\right)\left(\frac{-r_{ij}}{Q^2}\right)^{-\epsilon} + \left(\epsilon^{-2} + \frac{1}{2}\epsilon^{-1}\right)\underbrace{\left[\left(\frac{m_i^2}{Q^2}\right)^{-\epsilon} + \left(\frac{m_j^2}{Q^2}\right)^{-\epsilon}\right]}_{\text{dependence on fermion masses}}$$

Terms from $M_\mathrm{Z} \neq M_\mathrm{W}$:

$$\delta_{ij}^{\mathrm{Z}} = -\frac{\alpha}{4\pi}I_i^\mathrm{Z}I_j^\mathrm{Z}\,\ln\left(\frac{M_\mathrm{Z}^2}{M_\mathrm{W}^2}\right)\left(2L + 2L^2\epsilon + L^3\epsilon^2\right)$$

IV Summary & outlook

Massless and massive fermionic processes $f_1 f_2 \to f_3 \cdots f_n$

with $(p_i + p_j)^2 \gg M_\mathrm{W}^2$ and different masses $M_\mathrm{W}^2 \sim M_\mathrm{Z}^2 \sim m_\mathrm{t}^2 \sim M_\mathrm{Higgs}^2$:

- 2-loop EW NLL corrections in $D = 4 - 2\epsilon$ dimensions
  [$m_f = 0$: Denner, B.J., Pozzorini, Nucl. Phys. B 761 (2007) 1]
- factorizable contributions calculated with two independent methods:
  1.) sector decomposition, 2.) expansion by regions & Mellin–Barnes
- non-factorizable contributions vanish (collinear Ward identities)
- Yukawa contributions only in field strength renormalization
- universal correction factors, electromagnetic singularities separable
- applicable for $e^+e^- \to f\bar{f}$, Drell–Yan, . . .

Outlook: arbitrary processes

- generalize method for external gauge bosons & scalars
- calculate necessary loop integrals
- goal: process-independent 2-loop NLL corrections