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Electroweak contributions to $e^+e^- \longrightarrow W^+W^-b\bar{b}$ in the $t\bar{t}$ resonance region

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II Evaluation of electroweak NLO hard contributions
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I Top-pair production at linear colliders near threshold

Future linear colliders (ILC/CLIC)
with $\sqrt{s} \gtrsim 2m_t \approx 350\,\text{GeV} \rightsquigarrow$ produce $t\bar{t}$ pairs:
clean initial state of $e^+e^- \to t\bar{t}$ allows threshold scans with $\sqrt{s} \sim 2m_t$

$\hookrightarrow$ Precise determination of the top mass m_t, the width Γ_t, the Yukawa coupling λ_t
without the uncertainties/ambiguities of hadron colliders. Martinez, Miquel '02

Need also precise theoretical prediction
$\Rightarrow \delta\sigma/\sigma \sim$ 2–3% ($\delta\sigma \sim 5\,\text{fb}$ below threshold)

QCD corrections are known (almost) up to NNNLO order,
but electroweak (EW) NLO hard contributions are missing!

Also: decay $t\bar{t} \to (bW^+)\,(\bar{b}W^-)$ is an EW effect
$\Rightarrow$ describe $t\bar{t}$ production in terms of the more physical process $e^+e^- \to W^+W^-b\bar{b}$
and allow for invariant-mass cuts on reconstructed $t, \bar{t}$.

Perturbative expansion: NRQCD

Top quarks move slowly at threshold: $v = \sqrt{1 - \frac{4m_t^2}{s}} \sim \alpha_s \ll 1$

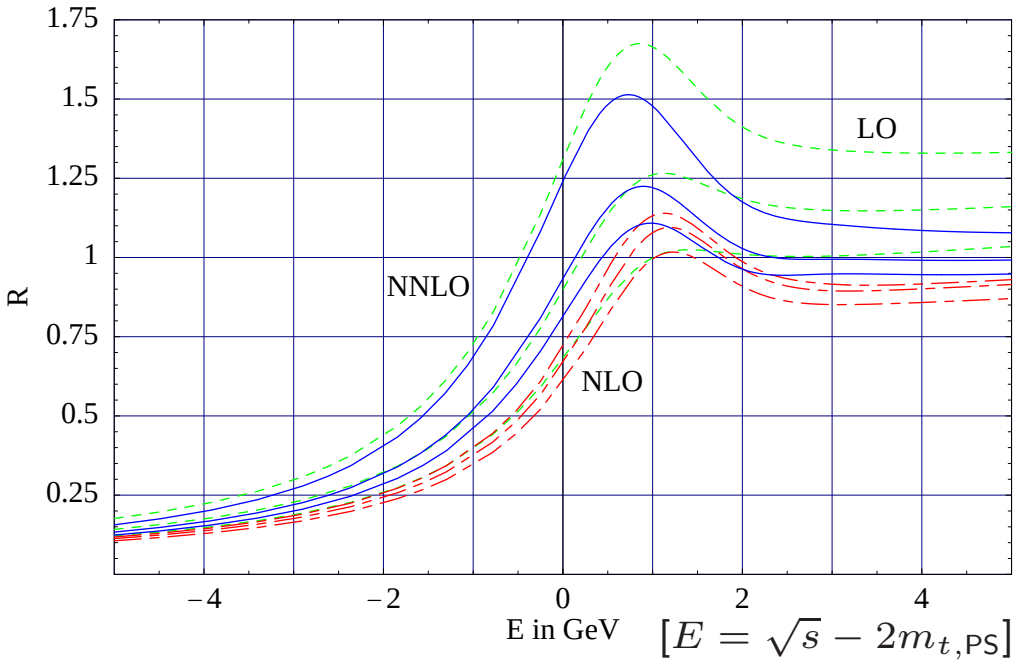
$\hookrightarrow$ sum $\left(\frac{\alpha_s}{v}\right)^n$ from "Coulomb gluons" to all orders:

$$R = \frac{\sigma_{t\bar{t}}}{\sigma_{\mu^+\mu^-}} = v \sum_n \left(\frac{\alpha_s}{v}\right)^n \left(\{1\}_{\text{LO}} + \{\alpha_s, v\}_{\text{NLO}} + \{\alpha_s^2, \alpha_s v, v^2\}_{\text{NNLO}} + \ldots\right)$$

Further RGE improvement by summing also $(\alpha_s \ln v)^m$ to all orders: LL, NLL, ...

Status of QCD corrections

- NNLO QCD corrections
  Hoang, Teubner '98–'99; Melnikov, Yelkhovsky '98; Yakovlev '98; Beneke, Signer, Smirnov '99 [see plot]; Nagano, Ota, Sumino '99; Penin, Pivovarov '98–'99
- NNLO & (partial) NNLL
  Hoang, Manohar, Stewart, Teubner '00–'01; Hoang '03; Pineda, Signer '06
- (partial) NNNLO
  Beneke, Kiyo, Schuller '05–'08 [+ contributions from Kiyo, Seidel, Steinhauser '08; Anzai, Kiyo, Sumino '09; Smirnov, Smirnov, Steinhauser '09–'10]



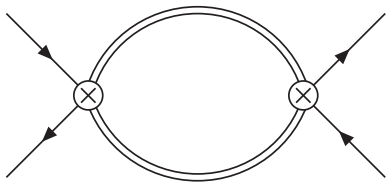
Effective field theory (EFT) for pair production of unstable particles near threshold

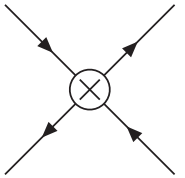
Beneke, Chapovsky, Khoze, Signer, Stirling, Zanderighi '01–'04;
Actis, Beneke, Falgari, Schwinn, Signer, Zanderighi '07–'08

- Non-relativistic power counting: $\boxed{\alpha_s^2 \sim \alpha_{\text{ew}} \sim \frac{\Gamma_t}{m_t} \sim v^2 \approx \delta = \frac{s}{4m_t^2} - 1}$
- Integrate out hard modes $\sim m_t$ $\rightsquigarrow$ EFT with potential (nearly on-shell) top quarks.
- Extract cross section $e^+e^- \to W^+W^-b\bar{b}$ from appropriate cuts of the $e^+e^- \to e^+e^-$ forward-scattering amplitude:

$$i\mathcal{A} = \underbrace{\sum_{k,l} \int \mathrm{d}^4x \, \langle e^+e^-|T\big[i\mathcal{O}_p^{(k)\dagger}(0)\, i\mathcal{O}_p^{(l)}(x)\big]|e^+e^-\rangle}_{\text{resonant contributions}} + \underbrace{\sum_k \langle e^+e^-|i\mathcal{O}_{4e}^{(k)}(0)|e^+e^-\rangle}_{\text{non-resonant contributions}}$$

with $t\bar{t}$ production operators $\mathcal{O}_p^{(k)}$





correspond to full-theory $e^+e^- \to e^+e^-$ with $\Gamma_t = 0$

$\Rightarrow$ Potential (+ soft ...) corrections to resonant diagrams within EFT

$\Rightarrow$ Hard corrections to matching coefficients of operators $\mathcal{O}_p^{(k)}$ and $\mathcal{O}_{4e}^{(k)}$.

Electroweak effects at LO

- Replacement rule $E = \sqrt{s} - 2m_t \to E + i\Gamma_t$ Fadin, Khoze '87

Electroweak effects at NLO

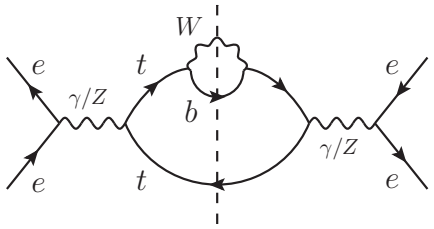
- Exchange of a "Coulomb photon": trivial extension of QCD corrections, available
- Gluon exchange between $t, \bar{t}$ and their decay products: these contributions cancel at NLO in the total cross section, Fadin, Khoze, Martin '94; Melnikov, Yakovlev '94
  they are negligible if the top invariant-mass cuts are loose enough.
- **Hard corrections** to the matching coefficient of the **non-resonant operator** $\mathcal{O}_{4e}^{(k)}$
  $\hookrightarrow$ *topic of this talk!*

The resonant **NNLO** corrections involve "finite-width divergences" $\propto \alpha_s \dfrac{\Gamma_t}{\epsilon}$ (in dim. reg.). These must be cancelled by EW NNLO hard contributions.
$\hookrightarrow$ Motivation for calculating EW hard corrections (starting at NLO ...).

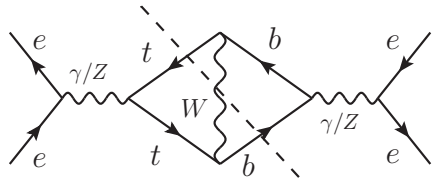
II Evaluation of electroweak NLO hard contributions

Hard corrections at NLO:

- cuts through $bW^+\bar{t}$ (see diagrams) and $\bar{b}W^-t$ (not shown) in the 2-loop forward-scattering amplitude
- correspond to tree-level processes $e^+e^- \to bW^+\bar{t}$ and $e^+e^- \to \bar{b}W^-t$
- expansion in $\delta = \frac{s}{4m_t^2} - 1$
  $\hookrightarrow$ at NLO: $s = 4m_t^2$



h1



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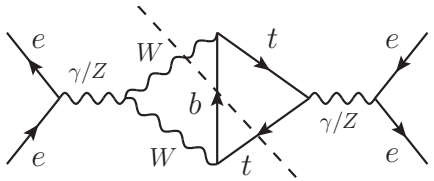

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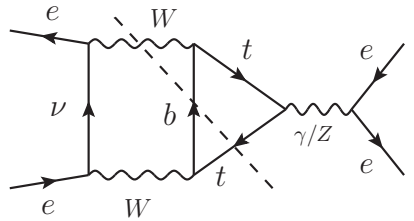

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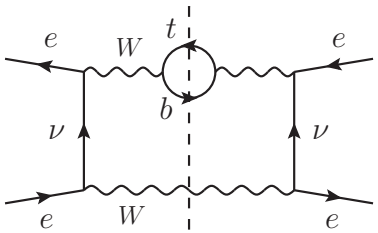

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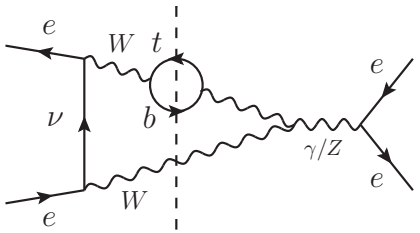

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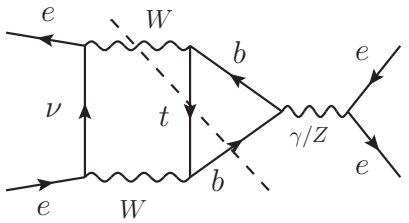

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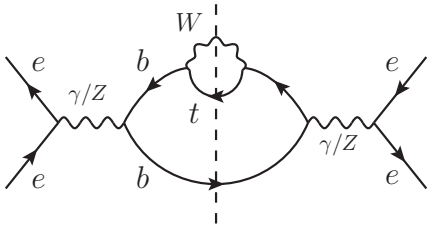

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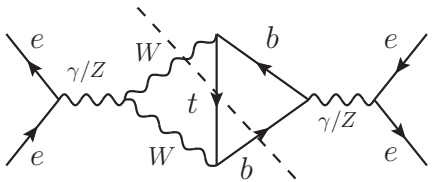
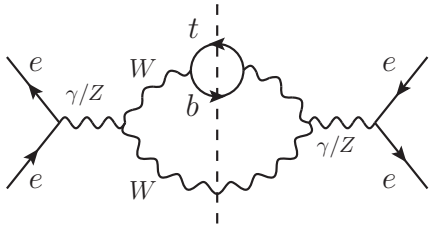


h9



h10

[symmetric diagrams not shown]

Form of hard contributions

With the reconstructed top momentum $p_t = p_b + p_{W^+}$, the contributions of diagrams h1–h10 (for $s = 4m_t^2$) are of the form:

$$H_i = \int_{\Delta^2}^{m_t^2} \mathrm{d}p_t^2 \, h_i\left(\frac{p_t^2}{m_t^2}, \frac{M_W^2}{m_t^2}\right)$$

with $\Delta^2 = M_W^2$ for the total cross section.

Invariant-mass cuts

Restrict invariant masses of the reconstructed $t, \bar{t}$: $\left|\sqrt{p_{t,\bar{t}}^2} - m_t\right| \leq \Delta M$

$\hookrightarrow$ hard contributions with $\Delta^2 = m_t^2 - \Lambda^2$ where $\Lambda^2 = (2m_t - \Delta M)\Delta M$.

We focus on loose cuts with $\Lambda^2 \gg m_t\Gamma_t$ (typical offshellness of potential top quarks), corresponding to $\Delta M \gg \Gamma_t \rightsquigarrow$ no cut needed for potential contributions.

In contrast: for tight cuts with $\Lambda^2 \lesssim m_t\Gamma_t$ or $\Delta M \lesssim \Gamma_t \rightsquigarrow$ hard contributions are absent and potential contributions need to be cut.

III Results & comparisons

EW NLO hard contributions: numeric integration over p_t^2 (and over one angle for some diagrams), the integrand is an analytic function of p_t^2/m_t^2 and M_W^2/m_t^2, the cut-dependence enters in the integration limit.

Parameters: $m_t = 172\,\text{GeV}$, $\Gamma_t = \Gamma_t^{\text{tree}} = 1.46550\,\text{GeV}$, on-shell (pole) masses, α and $\sin^2\theta_{\text{w}}$ from G_{F}, M_W, M_Z

Comparison to recent alternative approach (HRR) Hoang, Reißer, Ruiz-Femenía '10

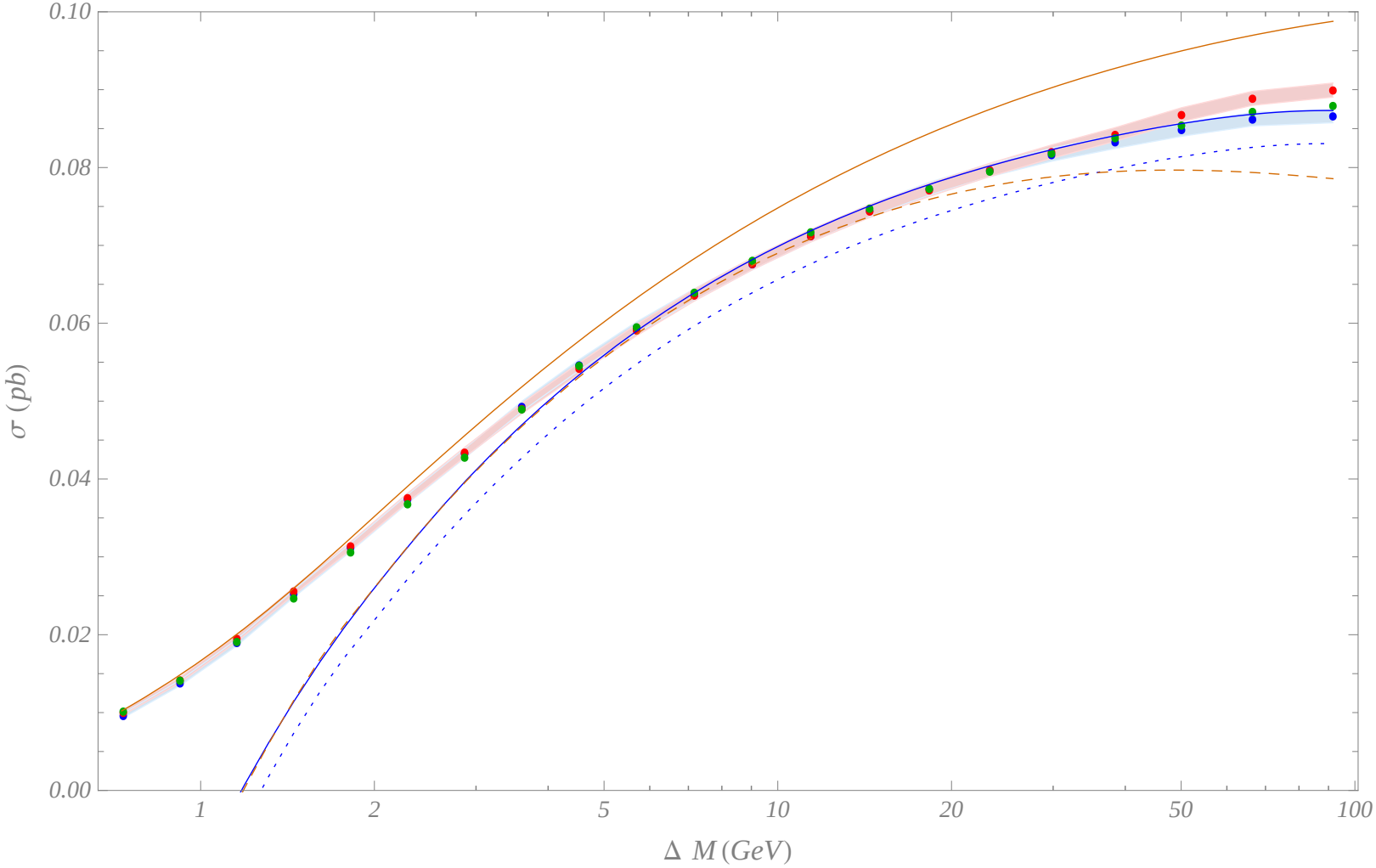
- Here QCD & EW contributions expanded for moderate invariant-mass cuts $15\,\text{GeV} \leq \Delta M \leq 35\,\text{GeV}$
  $\hookrightarrow$ our result is also valid for larger cuts up to the total cross section.
- EW contributions match expansion of our result in $(\Lambda/m_t)^n$ up to $n = 1$
- HRR had to neglect the non-t diagrams h5–h10, these "single-resonant" contributions are systematically included in our calculation.

Comparison to MadGraph/MadEvent/MadAnalysis (MG) Alwall et al. '07

$\hookrightarrow$ generated 10^4 events for $e^+e^- \to W^+W^-b\bar{b}$, analyzed cut-dependence

EW contributions: cut-dependence at threshold

cross section (for $\alpha_s = 0$) at threshold ($s = 4m_t^2$) as a function of the invariant-mass cut ΔM



total cross section reached for $\Delta M = 91.6\,\text{GeV}$

MG points (with statistical error bands): full, without Higgs, only t- or $\bar{t}$-diagrams

Our result: EW NLO hard + LO (dashed-blue) / NNLO (solid-blue) potential contributions

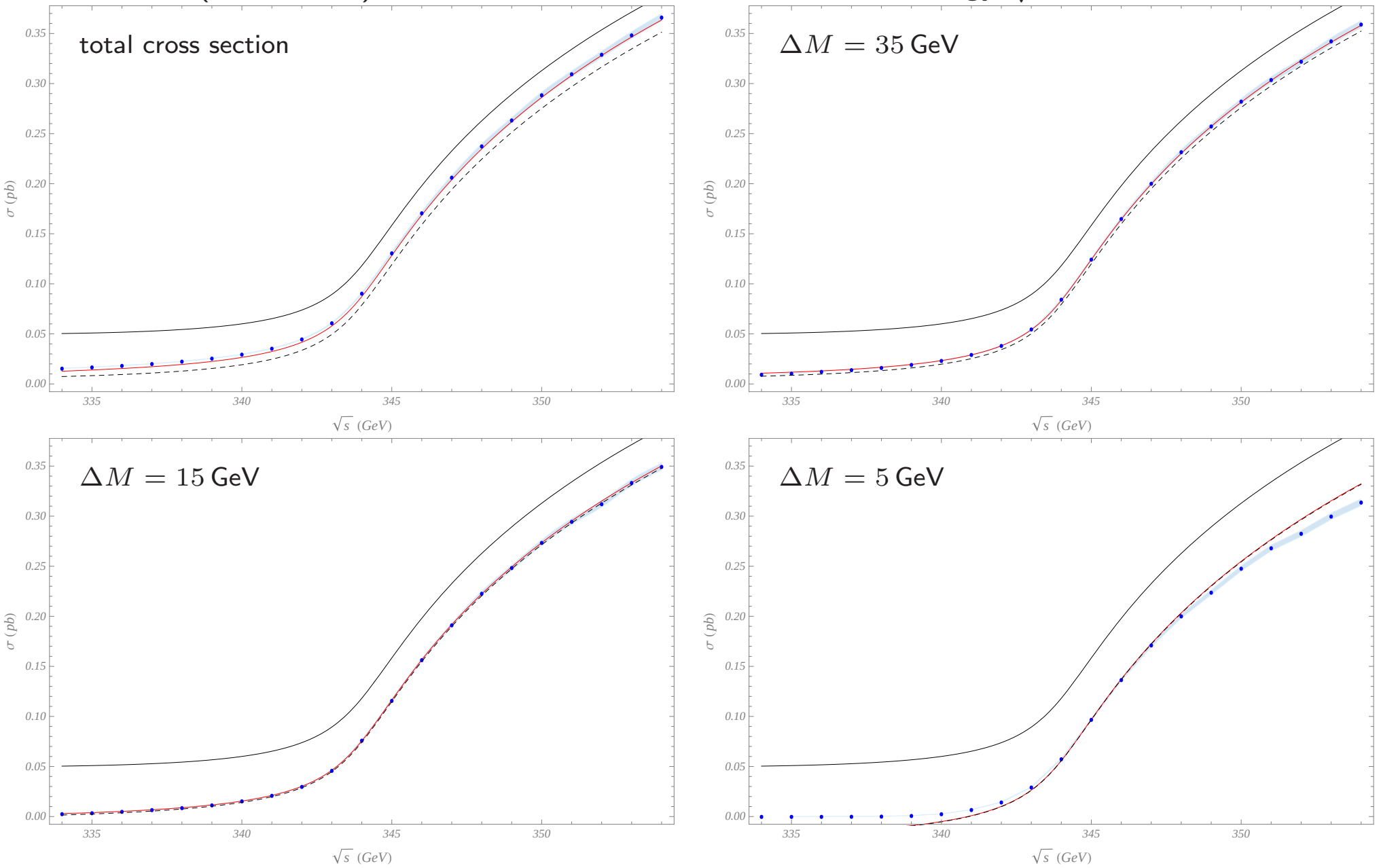
↪ good agreement with MG for loose cuts $\Delta M \gtrsim 5\,\text{GeV}$

Cut potential region (LO): solid-brown ⇒ good agreement with MG for tight cuts $\Delta M \lesssim 1\,\text{GeV}$

HRR result: dashed-brown ⇒ agrees with our result for small ΔM

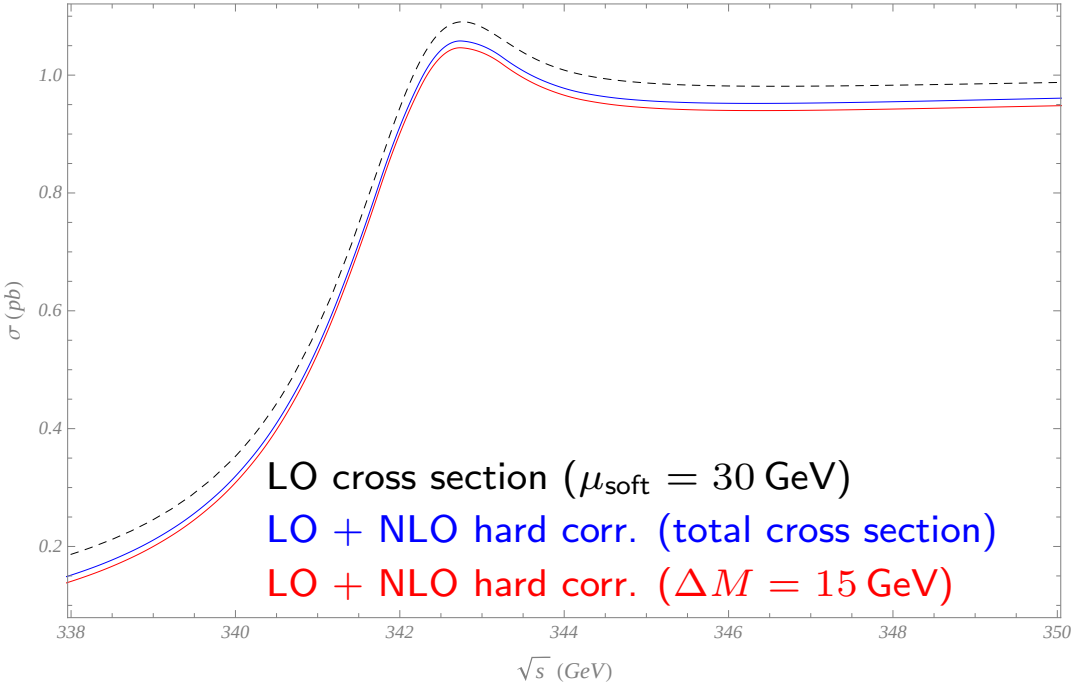
EW contributions: energy-dependence for different cuts

cross section (for $\alpha_s = 0$) as a function of the center-of-mass energy $\sqrt{s}$



MG (full) points & error band, our result, HRR result (dashed), only potential contributions (solid)

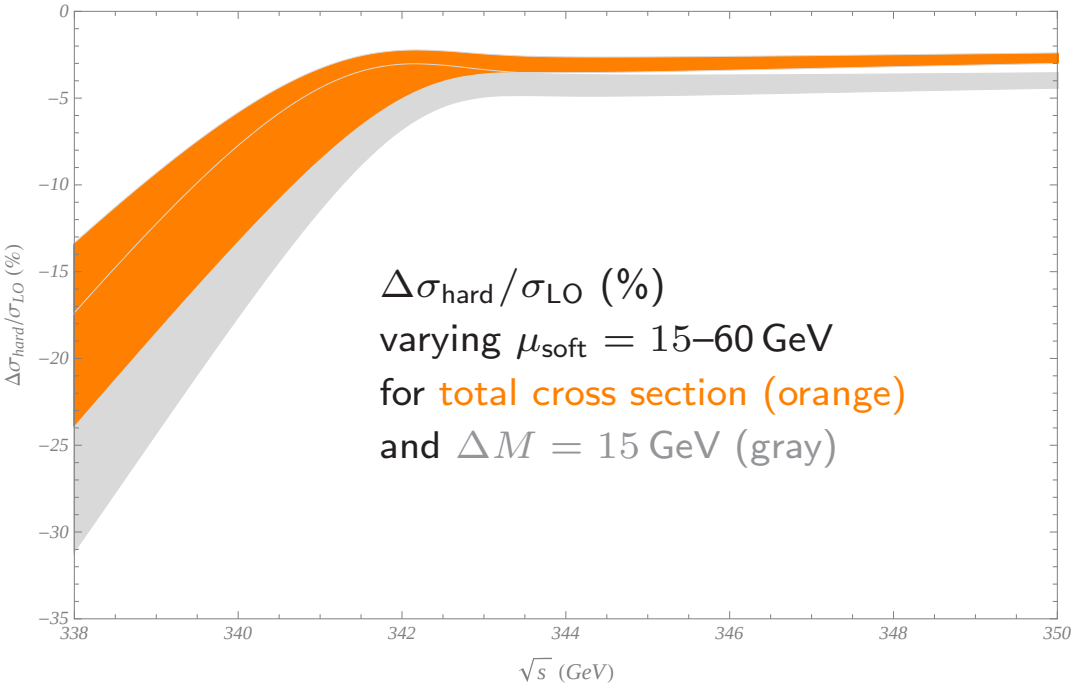
Full cross section with QCD & EW contributions



QCD contributions with $\alpha_s^{\overline{\text{MS}}}(\mu_{\text{soft}})$ (obtained from $\alpha_s^{\overline{\text{MS}}}(M_Z) = 0.118$)

LO cross section (potential QCD & EW contributions, dashed-black) and including our **EW NLO hard contributions** (solid-colored)

[NLO QCD contributions not shown]



Relative correction (in %) of EW NLO hard contribution w.r.t. LO cross section

IV Conclusions & outlook

Electroweak contributions to $e^+e^- \to W^+W^-b\bar{b}$ in the $t\bar{t}$ resonance region

- NLO contribution completed by EW non-resonant (hard) contributions
  for total cross section and with top invariant-mass cuts
- correction of $\sim$ -30 fb (-3% above and much more below threshold)
  for total cross section, even more with invariant-mass cuts
- good agreement with MadGraph for loose cuts
- good agreement with Hoang–Reißer–Ruiz-Femenía result for small cuts

$\hookrightarrow$ can be added to existing QCD results to improve accuracy of theoretical prediction

Future improvements

- add initial-state radiation and convolution with electron distribution functions
- evaluate leading EW NNLO contributions $\Rightarrow$ cancel finite-width divergences