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Electroweak Sudakov logarithms

The form factor in a massive U(1) model and in a U(1)$\times$U(1) model with mass gap

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I Why logarithmic 2-loop results in EW theory?

Electroweak (EW) precision physics

- experimentally measured by now at energy scales up to $\sim M_{W,Z}$
- future generation of accelerators (LHC, LC) $\rightarrow$ TeV region

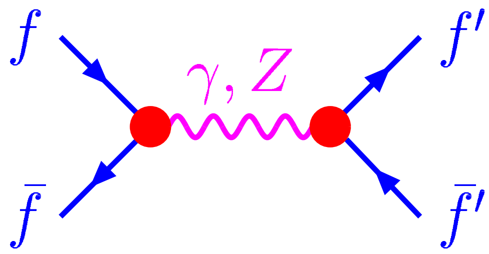
Electroweak radiative corrections at high energies $\sqrt{s} \sim$ TeV $\gg M_{W,Z}$

Kühn et al. '00, '01; Fadin et al. '00; Denner et al. '01, '03; B.F. et al. '03; Pozzorini '04; ...

large negative corrections in *exclusive* cross sections

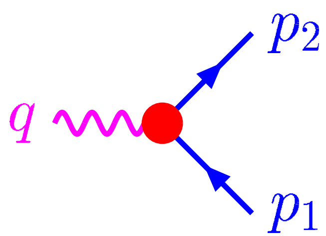
- EW corrections dominated by Sudakov logarithms $\alpha^n \ln^{2n}(s/M_{W,Z}^2)$
- 1-loop corrections $\gtrsim 10\%$
- 2-loop corrections $\sim 1\%$, need to be under control for LC

Important class of processes: 4-fermion scattering



$$= \frac{ig^2}{s} F^2 \tilde{A}$$

Form factor F of vector current:



$$= \bar{u}(p_2)\, \gamma^\mu\, u(p_1) \cdot F + \ldots$$

High energy behaviour → *Sudakov limit*

- momentum transfer $-q^2 \equiv Q^2 \gg M^2 \equiv M^2_{W,Z}$
- neglect fermion masses
- *logarithmic approximation*: neglect terms suppressed by a factor of M^2/Q^2

  ↪ works well for 2-loop n_f contribution where the exact result in M^2/Q^2 is known

  B.F., Kühn, Moch '03

Simplified models

1. Decompose the problem into simpler parts:

Standard Model
SU(2)$_\mathrm{L}$ $\otimes$ U(1)
$M_W \neq M_Z$, $M_\gamma = 0$

$\longleftrightarrow$

U(1)$_M$ $\otimes$ U(1)$_\lambda$ model
2 *Abelian* interactions
mass gap $M \gg \lambda \to 0$

$\longleftrightarrow$

massive U(1) model
Abelian ($\sim$ QED)
gauge boson mass M

$\updownarrow$ (U(1)$_M$ $\otimes$ U(1)$_\lambda$ model ↔ SU(2) $\otimes$ U(1)$_\lambda$ model)

$\updownarrow$ (massive U(1) model ↔ massive SU(2) model)

$\hookrightarrow$

SU(2) $\otimes$ U(1)$_\lambda$ model
$M_W \approx M_Z \gg \lambda \to 0$

$\longleftrightarrow$

massive SU(2) model
gauge boson mass M

2. Use the partial results to compose a precise approximation of the Standard Model result.

II Massive U(1) form factor

Form factor in perturbation theory: $F = 1 + \alpha\, F_1 + \alpha^2\, F_2 + \ldots$

large radiative corrections for $Q \sim$ TeV $\rightarrow$ sum up large logarithms to all orders in α

Evolution equation in logarithmic approximation: Sen '81; Collins '89; Korchemsky '89; ...

$$\frac{\partial F(Q^2)}{\partial \ln Q^2} = \left[\int_{M^2}^{Q^2} \frac{dx}{x} \gamma\big(\alpha(x)\big) + \zeta\big(\alpha(Q^2)\big) + \xi\big(\alpha(M^2)\big)\right] F(Q^2)$$

Solution $\rightarrow$ exponentiation:

$$F(Q^2) = F_0\big(\alpha(M^2)\big) \exp\left\{\int_{M^2}^{Q^2} \frac{dx}{x} \left[\int_{M^2}^{x} \frac{dx'}{x'} \gamma\big(\alpha(x')\big) + \zeta\big(\alpha(x)\big) + \xi\big(\alpha(M^2)\big)\right]\right\}$$

$\Rightarrow$ Resummation:

$$F = 1 + \alpha\,(\ln^2 + \ln + \text{const}) + \alpha^2\,(\ln^4 + \ln^3 + \ln^2 + \ln + \text{const}) + \ldots$$

$$\leftrightarrow \big(1 + \alpha \cdot \text{const} + \alpha^2 \cdot \text{const} + \ldots\big) \exp\Big(\alpha\,(\ln^2 + \ln) + \alpha^2\,(\ln^3 + \ln^2 + \ln) + \ldots\Big)$$

Massive U(1) form factor in 2-loop approximation

Known from resummation & full calculation of n_f contribution: ($n_f = \#$ fermions)

$$\alpha^2 F_2 = \left(\frac{\alpha}{4\pi}\right)^2 \left[+\frac{1}{2}\ln^4\left(\frac{Q^2}{M^2}\right) - \left(\frac{4}{9}n_f + 3\right)\ln^3\left(\frac{Q^2}{M^2}\right) + \left(\frac{38}{9}n_f + \frac{2}{3}\pi^2 + 8\right)\ln^2\left(\frac{Q^2}{M^2}\right) - \left(\frac{34}{3}n_f + \ldots\right)\ln\left(\frac{Q^2}{M^2}\right) + \left(\frac{16}{27}\pi^2 + \frac{115}{9}\right)n_f + \ldots \right]$$

Kühn, Moch, Penin, Smirnov '01
B.F., Kühn, Moch '03

- growing coefficients with alternating sign:

$$\begin{array}{cccccccc} & - 0.4\, n_f \ln^3 & + & 4.2\, n_f \ln^2 & - & 11.3\, n_f \ln & + & 18.6\, n_f \\ + 0.5 \ln^4 & - \quad 3 \quad \ln^3 & + & 14.6 \ \ln^2 & - & \ldots \quad \ln & + & \ldots \end{array}$$

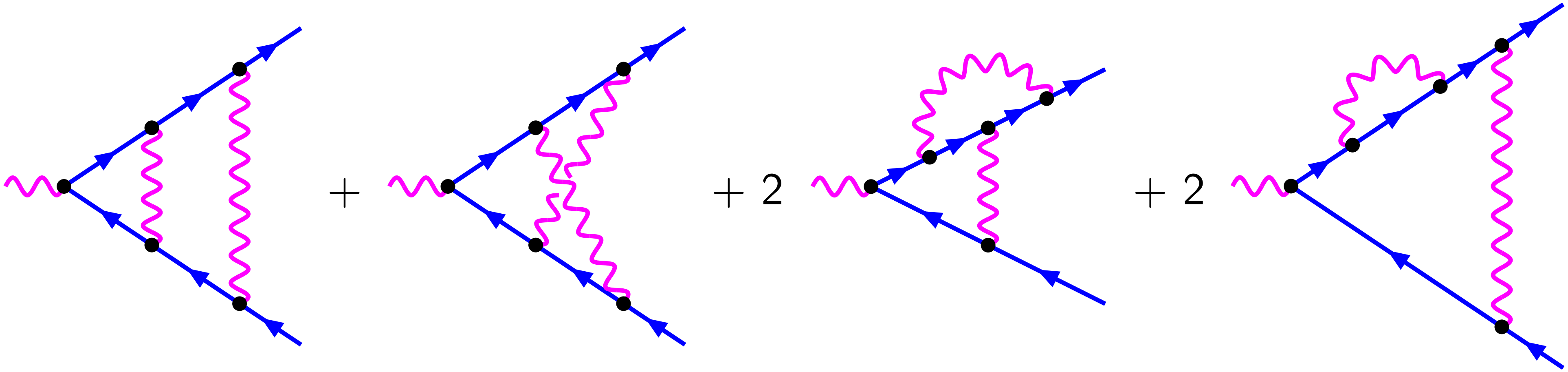
- $Q \sim 1\,\text{TeV} \to +\ln^4 \sim -\ln^3 \sim +\ln^2$

  $\to$ large cancellations between logarithmic terms

Complete 2-loop corrections in logarithmic approximation necessary.

Massive U(1) form factor in 2-loop approximation: calculation $(n_f = 0)$

- complete 2-loop result $\longrightarrow$ loop calculation (*independent* of evolution equation)
- 2-loop vertex diagrams (massless fermions, massive bosons, 1 external scale):



$+$ external leg corrections $+$ 1-loop $\times$ 1-loop

- reduction to scalar diagrams $\longrightarrow$ FORM (Vermaseren)
- scalar diagrams: expansion by regions — Beneke, Smirnov '97
- evaluation of integrals and expansion in $\varepsilon = (4-d)/2 \longrightarrow$ Mathematica

Massive U(1) form factor in 2-loop approximation: result $(n_f = 0)$

B.F., Kühn, Penin, Smirnov, hep-ph/0404082

$$\alpha^2 F_2 = \left(\frac{\alpha}{4\pi}\right)^2 \Bigg[$$

$$+\frac{1}{2}\ln^4\left(\frac{Q^2}{M^2}\right)$$

$$-3\ln^3\left(\frac{Q^2}{M^2}\right)$$

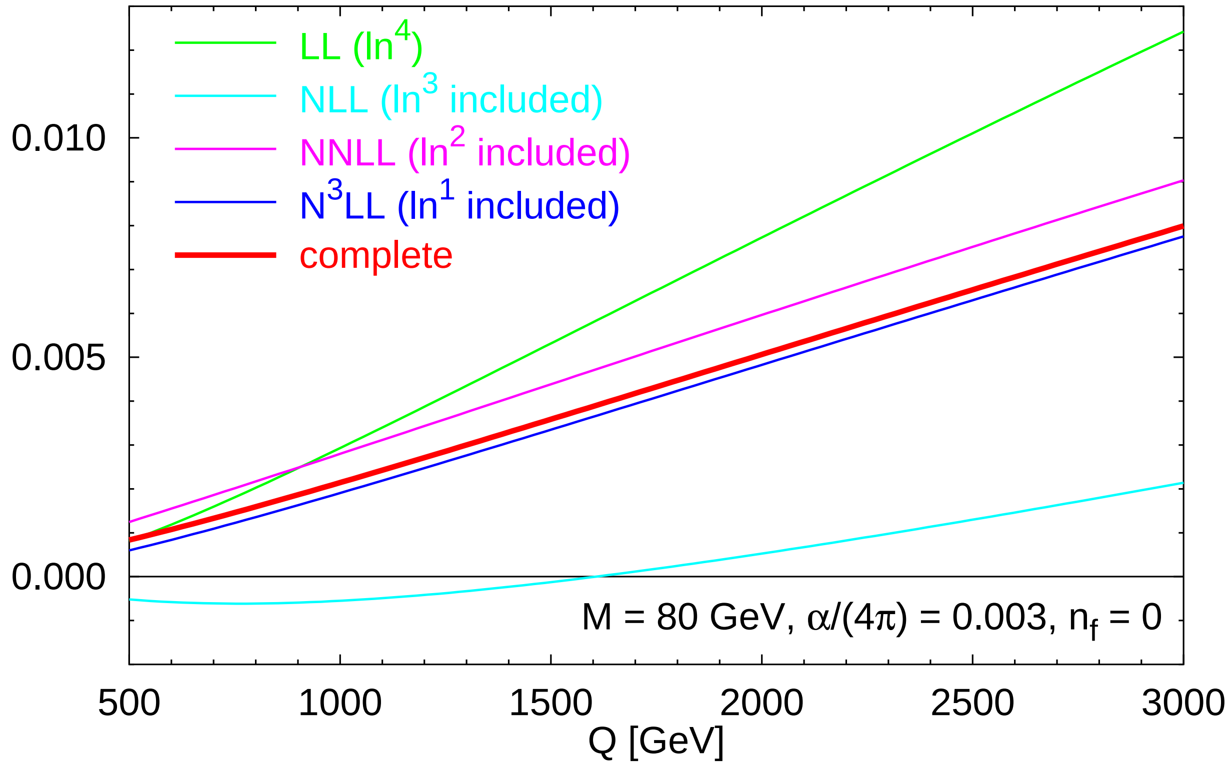
$$+\left(\frac{2}{3}\pi^2+8\right)\ln^2\left(\frac{Q^2}{M^2}\right)$$

agreement ✓

$$-\left(-24\zeta_3+4\pi^2+9\right)\ln\left(\frac{Q^2}{M^2}\right)$$

$$+256\,\mathrm{Li}_4\left(\frac{1}{2}\right)+\frac{32}{3}\ln^4 2-\frac{32}{3}\pi^2\ln^2 2-\frac{52}{15}\pi^4+80\zeta_3+\frac{52}{3}\pi^2+\frac{25}{2}\Bigg]$$

new!



size of coefficients: $+0.5\ln^4 - 3\ln^3 + 14.6\ln^2 - 19.6\ln + 26.4$

at $Q = 1$ TeV: $+326 \quad -387 \quad +372 \quad -99.2 \quad +26.4$

$\Rightarrow$ alternating signs! small constant (N^{4}LL) contribution

III U(1)×U(1) model with mass gap

EW theory: massive and massless gauge bosons

↪ consider $\mathrm{U(1)}_M \times \mathrm{U(1)}_\lambda$ model with 2 different masses $M \gg \lambda \to 0$

- pure $\mathrm{U(1)}_M$: form factor $F(\alpha, Q, M)$
- pure $\mathrm{U(1)}_\lambda$: form factor $F(\alpha', Q, \lambda)$
  → known from massive U(1) result ($M \to \lambda$, $\alpha \to \alpha'$)
  → IR (soft/collinear) singularities regularized by λ (or by poles in ε if $\lambda = 0$)
- combined $\mathrm{U(1)}_M \times \mathrm{U(1)}_\lambda$: $\hat{F}(\alpha, \alpha', Q, M, \lambda)$
  $Q \gg M \gg \lambda \to$ **Factorization of IR singularities:**

$$\hat{F}(\alpha, \alpha', Q, M, \lambda) = \underbrace{F(\alpha', Q, \lambda)}_{\text{IR singular}} \underbrace{\tilde{F}(\alpha, \alpha', Q, M)}_{\text{IR finite}} + \mathcal{O}\left(\alpha\alpha' \frac{\lambda^2}{M^2}\right)$$

Factorization of U(1)×U(1) form factor: results $(n_f = 0)$

$$\hat{F}(\alpha,\alpha',Q,M,\lambda) = F(\alpha',Q,\lambda)\,\tilde{F}(\alpha,\alpha',Q,M) + \mathcal{O}\left(\alpha\alpha'\frac{\lambda^2}{M^2}\right)$$

$$\Rightarrow \tilde{F}(\alpha,\alpha',Q,M) = \lim_{\lambda\to 0}\frac{\hat{F}(\alpha,\alpha',Q,M,\lambda)}{F(\alpha',Q,\lambda)} = \lim_{\varepsilon\to 0}\frac{\hat{F}_\varepsilon(\alpha,\alpha',Q,M,0)}{F_\varepsilon(\alpha',Q,0)}$$

$\hookrightarrow$ set $\lambda = 0$ and calculate $\hat{F}_\varepsilon(\alpha,\alpha',Q,M,0)$ in dimensional regularization

Calculation of 2-loop diagrams with 1 massive and 1 massless gauge boson $\longrightarrow$

$$\tilde{F}(\alpha,\alpha',Q,M) = F(\alpha,Q,M)\,\times$$

$$\left\{1 + \frac{\alpha\alpha'}{(4\pi)^2}\left[\underbrace{\left(48\zeta_3 - 4\pi^2 + 3\right)}_{21.2}\ln\left(\frac{Q^2}{M^2}\right) + \underbrace{\frac{7}{45}\pi^4 - 84\zeta_3 + \frac{20}{3}\pi^2 - 2}_{-22.0}\right]\right\}$$

$\Rightarrow$ interference terms are finite $\rightsquigarrow$ IR singularities factorize

$\Rightarrow$ additional terms contain only single logarithm $\ln^1$

Factorization of U(1)×U(1) form factor for $\lambda = M$

$$\hat{F}(\alpha,\alpha',Q,M,\lambda) = F(\alpha',Q,\lambda)\,\tilde{F}(\alpha,\alpha',Q,M) + \mathcal{O}\left(\alpha\alpha'\frac{\lambda^2}{M^2}\right)$$

Form of the suppressed interference terms $\mathcal{O}\left(\alpha\alpha'\frac{\lambda^2}{M^2}\right)$?

↪ set $\lambda = M$ and parameterize:

$$\hat{F}(\alpha,\alpha',Q,M,M) = F(\alpha',Q,M)\,\tilde{F}(\alpha,\alpha',Q,M)\,C(\alpha,\alpha',Q,M)$$

On the other hand: $\hat{F}(\alpha,\alpha',Q,M,M) = F(\alpha+\alpha',Q,M)$

↪ known from massive U(1) result → calculate matching coefficient:

$$C(\alpha,\alpha',Q,M) = 1 + \frac{\alpha\alpha'}{(4\pi)^2}\underbrace{\left[512\,\mathrm{Li}_4\left(\tfrac{1}{2}\right) + \frac{64}{3}\ln^4 2 - \frac{64}{3}\pi^2\ln^2 2 - \frac{113}{15}\pi^4 + 244\zeta_3 + \frac{70}{3}\pi^2 + \frac{59}{4}\right]}_{-26.8}$$

⇒ interference term is constant, no logarithm

⇒ product $F(\alpha',Q,\lambda)\,\tilde{F}(\alpha,\alpha',Q,M)$ approaches $\hat{F}(\alpha,\alpha',Q,M,M)$ continuously for $\lambda \to M$ with N^3LL accuracy!

IV Applications

U(1)×U(1) form factor with mass gap from 1-mass result

Massive W, Z & massless photon $\longrightarrow$ need form factor with mass gap.

Suppose we cannot calculate $\hat{F}(\alpha, \alpha', Q, M, \lambda \to 0)$,
but we know $F(\alpha, Q, M)$ and $F(\alpha', Q, \lambda \to 0)$.

$\hookrightarrow$ Use $F(\alpha + \alpha', Q, M) = F(\alpha', Q, M)\, \tilde{F}(\alpha, \alpha', Q, M) + \mathcal{O}\big(\alpha\alpha' \ln^0\big)$

So we can get all logarithms in 2 loops:

$$\hat{F}(\alpha, \alpha', Q, M, \lambda \to 0) = F(\alpha', Q, \lambda \to 0)\, \frac{F(\alpha + \alpha', Q, M)}{F(\alpha', Q, M)} + \mathcal{O}\big(\alpha\alpha' \ln^0\big)$$

$\Rightarrow$ The calculation is reduced to the 1-mass case (with photon as heavy as W, Z).

<u>Note:</u>
SU(2)×U(1) model with mass gap $\longrightarrow$ result only up to $\mathcal{O}\big(\alpha\alpha' \ln^1\big)$

Expanding the U(1)×U(1) form factor in a small mass difference

Up to now, all heavy gauge bosons → same mass M.

But we need also $M_W \approx M_Z \to \lambda \approx M$:

$$\hat{F}(\alpha,\alpha',Q,M,\lambda) = F(\alpha',Q,\lambda)\,\tilde{F}(\alpha,\alpha',Q,M) + \underbrace{\mathcal{O}\left(\alpha\alpha'\frac{\lambda^2}{M^2}\right)}_{\mathcal{O}\left(\alpha\alpha'\ln^{0,1}\right),\ \lambda\to M}$$

↪ expand first term in $\delta \equiv \dfrac{M-\lambda}{M}$ for $\lambda \approx M$:

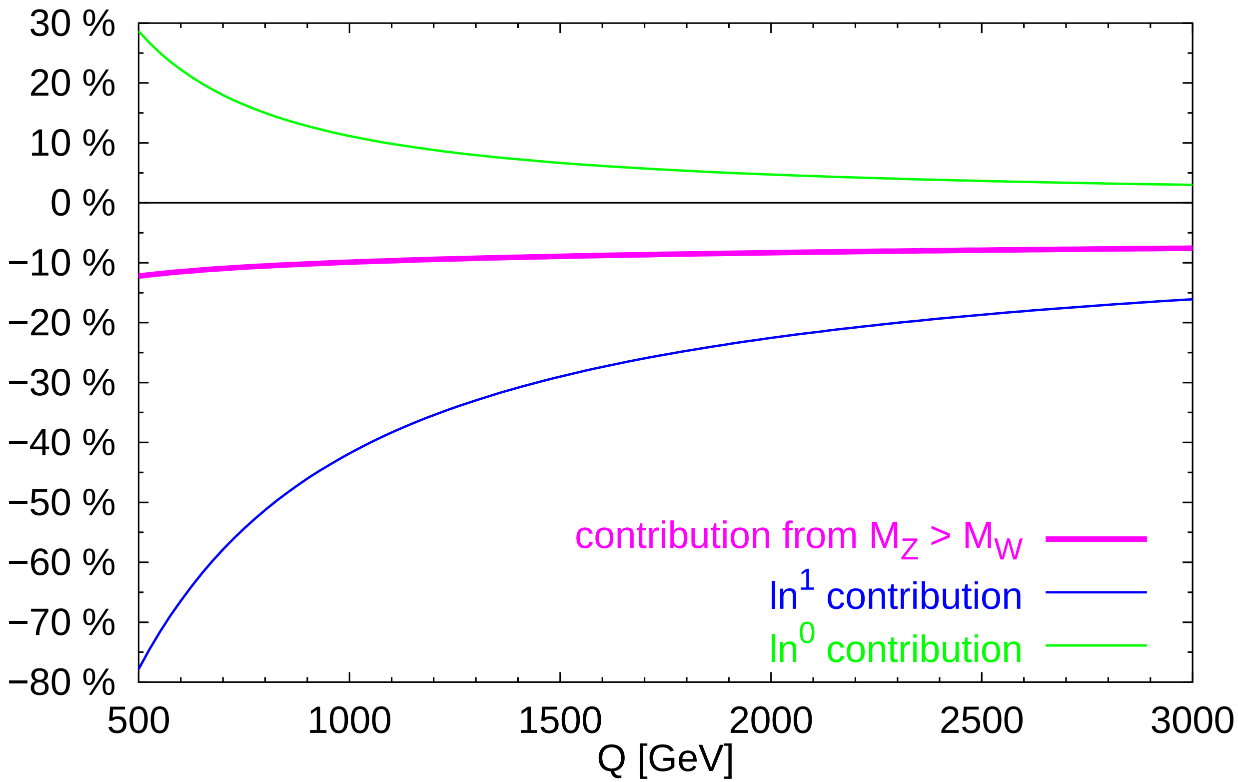
$$\hat{F}(\alpha,\alpha',Q,M,\lambda)\Big|_{\lambda\approx M} = F(\alpha+\alpha',Q,M)\cdot\left\{1-\delta\frac{\alpha'}{4\pi}\left[4\ln\left(\frac{Q^2}{M^2}\right)-6\right]+\mathcal{O}(\delta^2)\right\}$$
$$+\,\mathcal{O}\left(\delta\,\alpha\alpha'\ln^{0,1}\right)$$

Contribution of the $M_Z \neq M_W$ mass difference to the 2-loop form factor

$M_W = 80.4\,\text{GeV}$

$M_Z = 91.2\,\text{GeV}$

Relative contribution (in %)
of the mass difference $M_Z \neq M_W$
to the 2-loop form factor F_2



For comparison:

in blue/green: relative contribution of the linear logarithm / constant terms in F_2

$\Rightarrow$ The $M_Z \neq M_W$ mass difference can be taken into account
by an expansion around the equal mass approximation.

V Summary & outlook

Massive U(1) form factor

- complete 2-loop result in logarithmic approximation ✓

⇒ precise control of radiative corrections

U(1)×U(1) model with mass gap

- factorization of IR singularities shown explicitly ✓

Applications

- calculation with mass gap reduced to the 1-mass case $M_W = M_Z = M_{\text{photon}}$
- $M_Z \neq M_W$ taken into account by expanding around the equal mass approximation

Outlook

- extend to non-Abelian models: SU(2), SU(N), SU(2)×U(1)
- consider Higgs contributions
- 4-fermion scattering amplitude
- predictions for EW corrections to $f\bar{f} \to f'\bar{f}'$ cross sections